Course: Groundwater Flow and Cont Transp. Modeling
(for HG, EE, PGE students)
2019

Basics of theory of modeling

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What is a model?

- The model is the downscaled version of reality
  - (dummy or mock-up vs. model problem)
  - downscaling not only in size but also in all the important properties is needed
- The modell is a simplified and/or schematic transformation of the reality
  - too scientific and not really relevant
- The best I know: The modell completely fits at least one behavior of the reality
- Reality Model

Consequences of „model-being”

- The modell is task oriented (It is formulated to solve a given task (problem)
- The model fits only one or only some properties and behavior of the real systems
- We may need as many models of the real systems, as manydifferent tasks we want to deal with
-

Reality

Model1

Model2

Model3

Other consequences of „model-being”

- There is no universal or general model !!!

- The model is OK (good) in case it gives an accurate answer to the given problem

- From the sets of good models the best is the simpliest...

- There are two types of models:
  - good models
  - instructive models

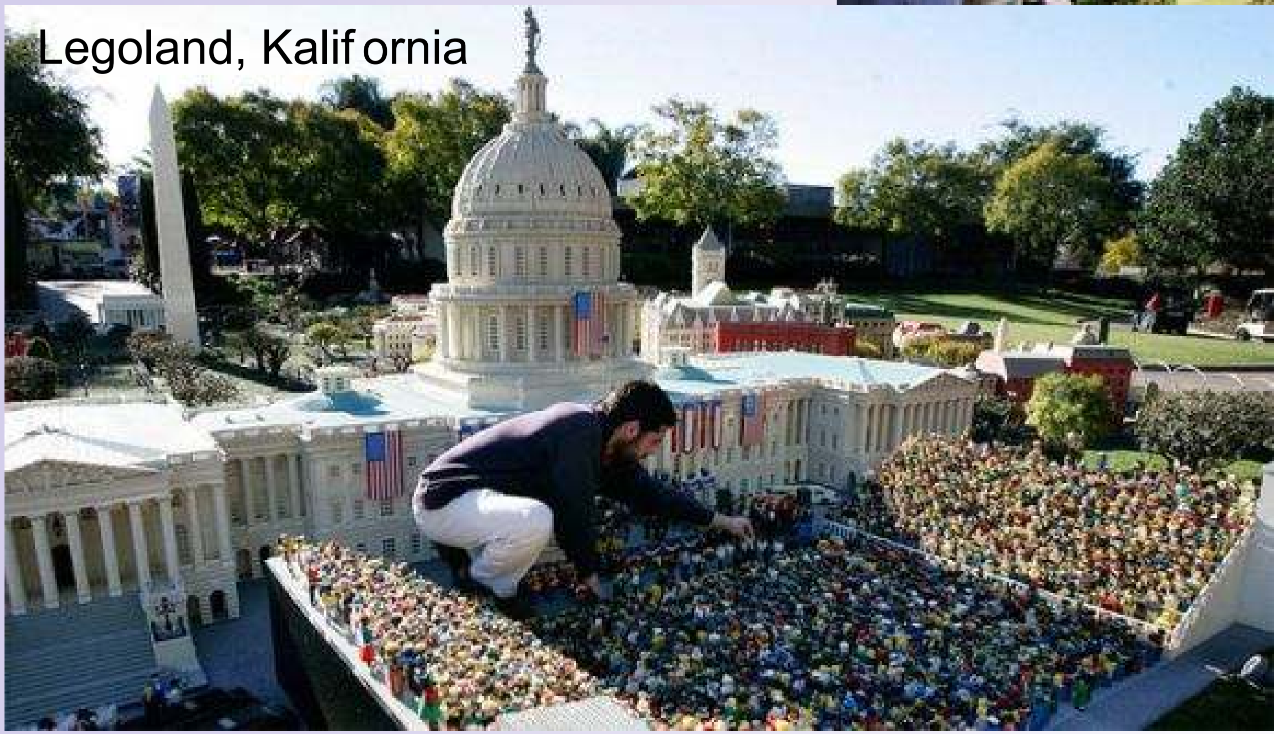
  - You must learn from all model runs
    - to understand the meaning of governing equations
    - to understand the real world processes
    - to determine the relevant and non-relevant properties of real systems
    - etc.

„All the models are wrong, but some are useful...”
(Prof. Jacob Bear)

Model types #1

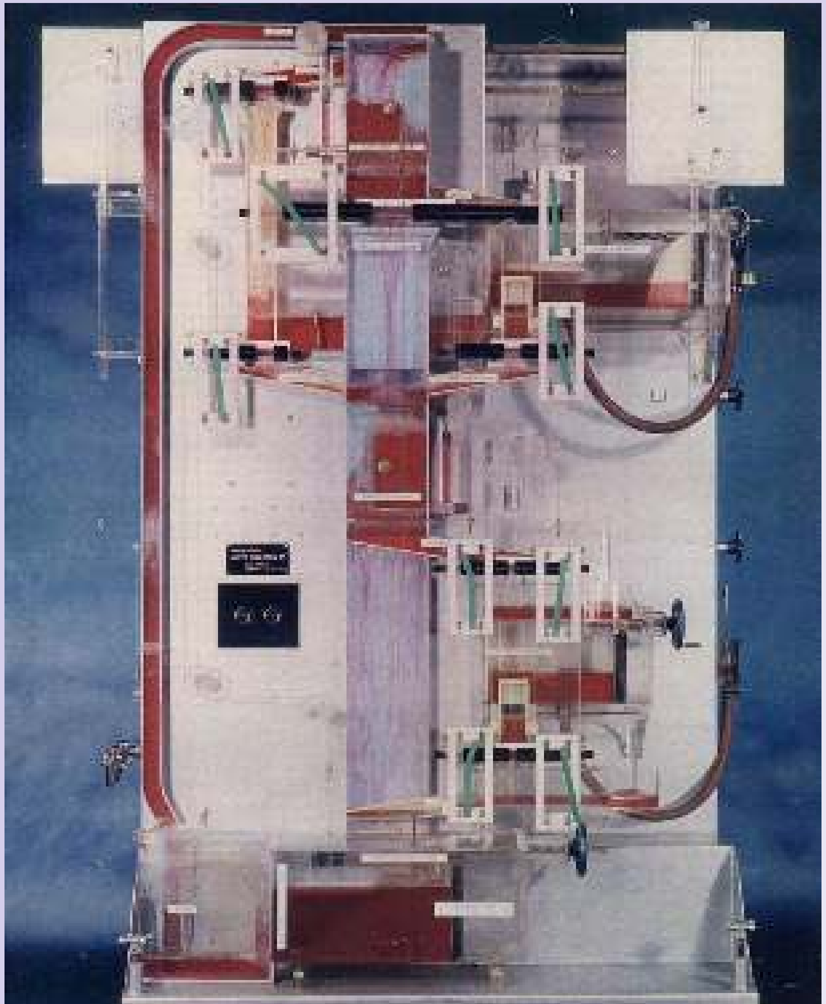
Physical model

- only downscaling

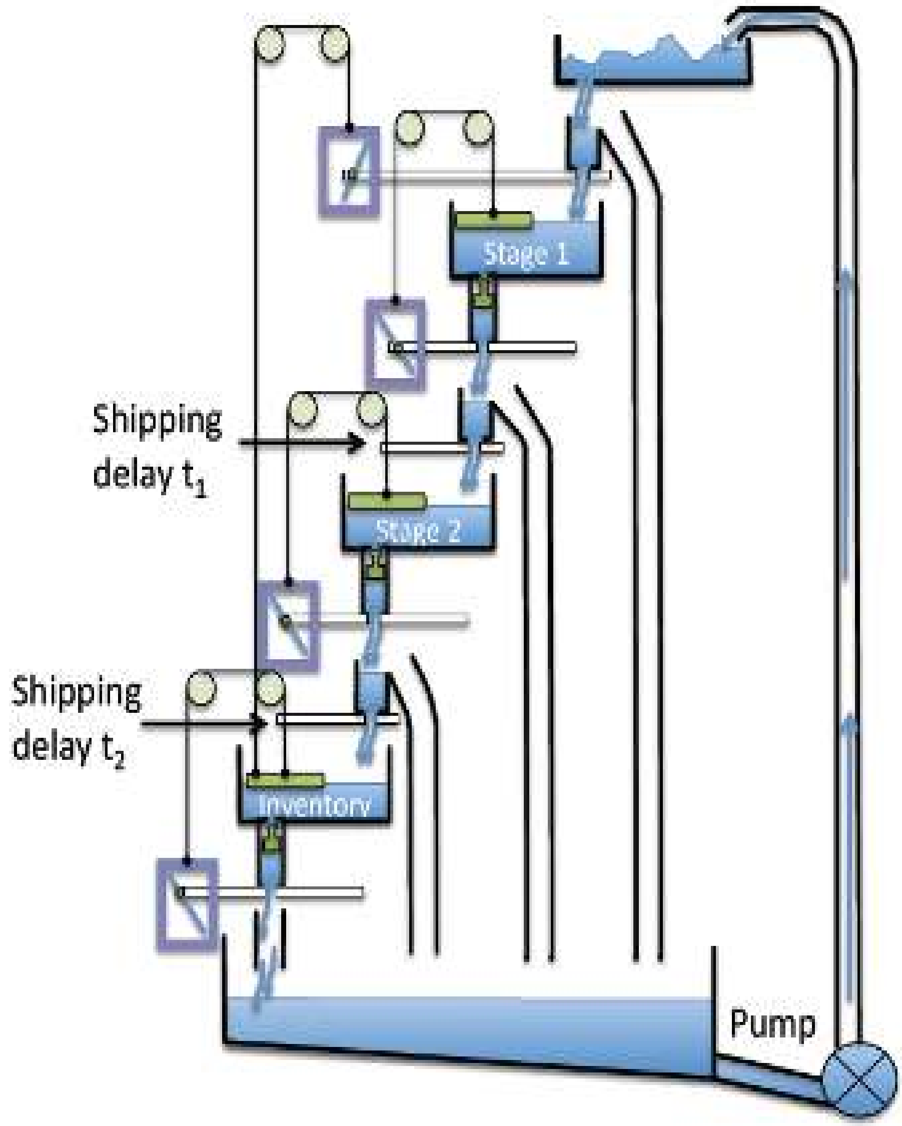


Model types #2

Analogue model:



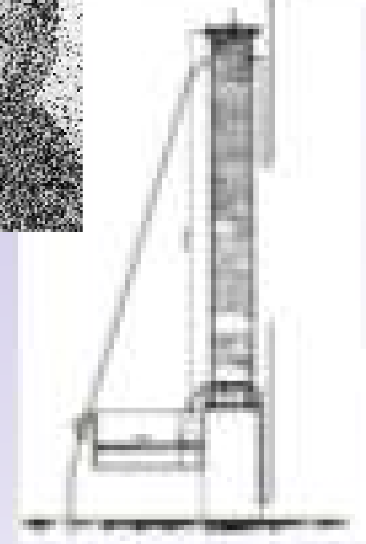
(W. Vandivert, Fortune, 1952)



(W.H.Ryder, 2008)

Model types #3

$$Q = -K \cdot A \frac{dh}{L}$$

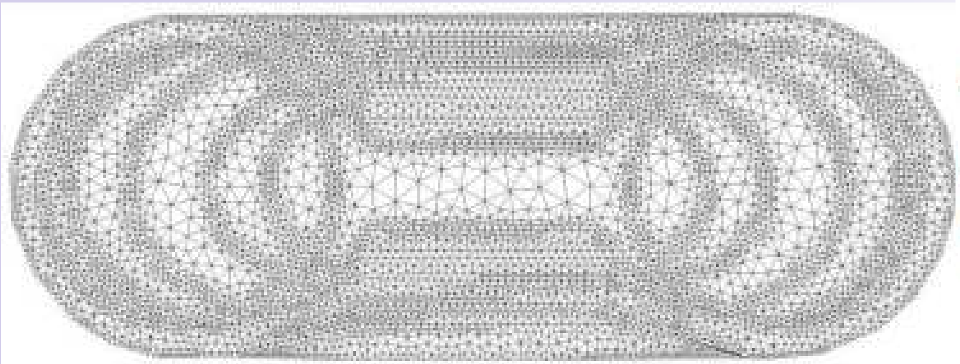


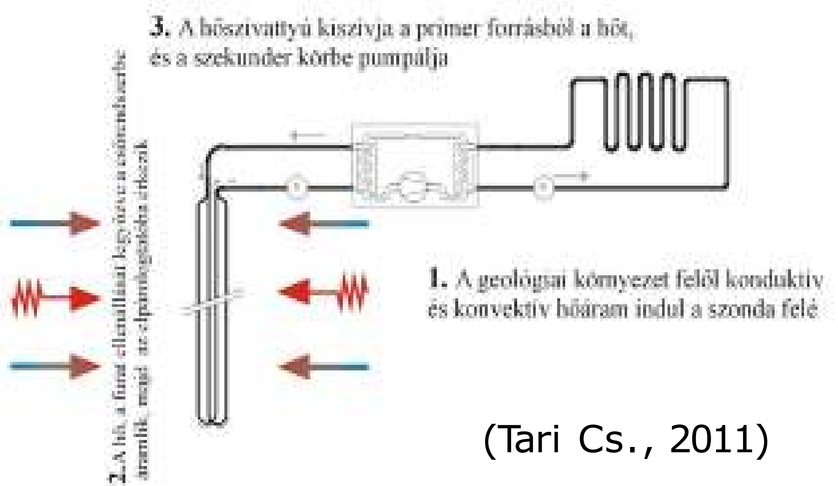
Freeze, R. A. (1994), Henry Darcy and the Fountains of Dijon. Ground Water, 32: 23–30.

Matematic models:

a felszín alatti vízáramlást leíró egyenletek megoldása (szivárgás alapegyenlete)

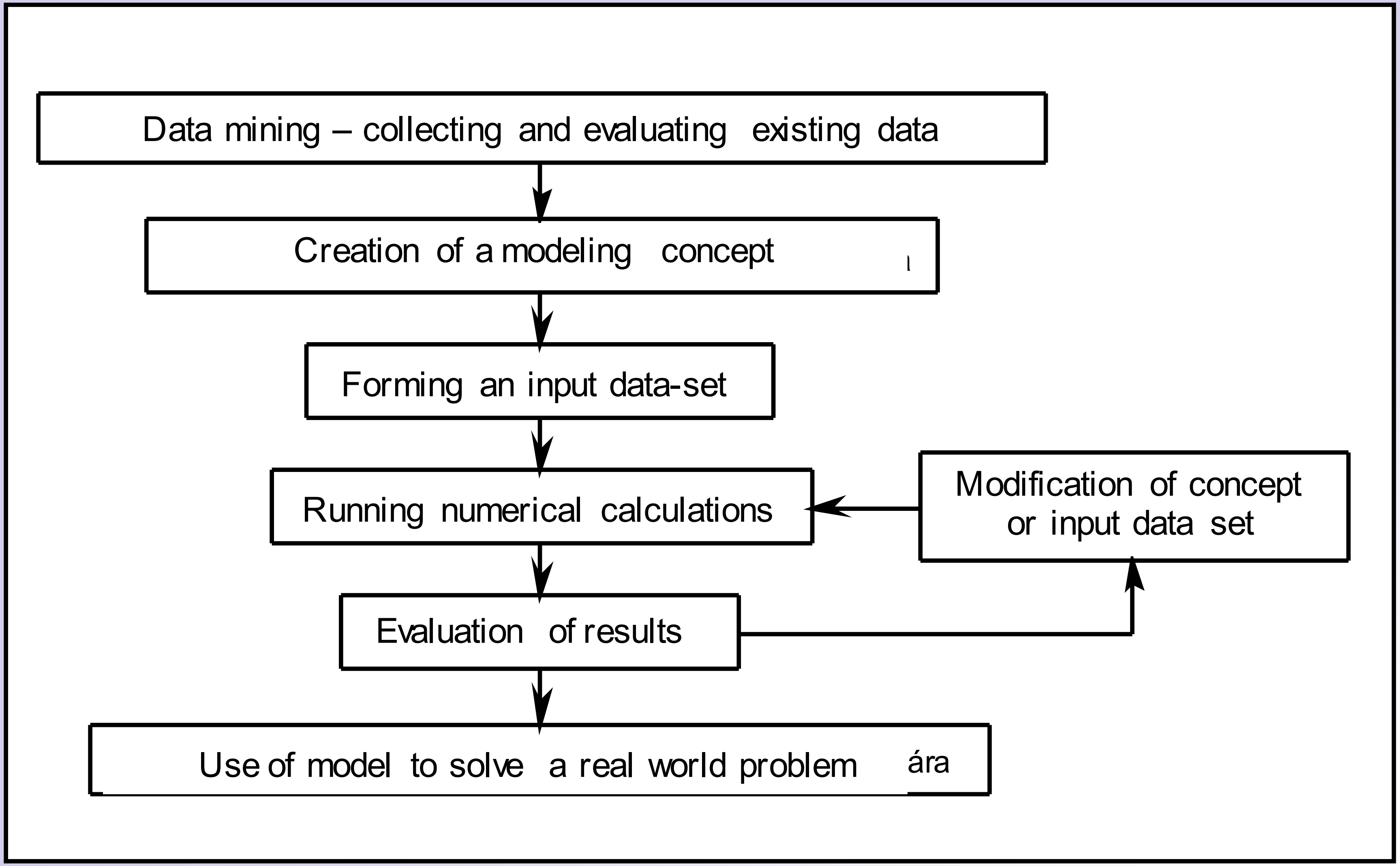
- **analitical models**: egzact, matematical solutions, but homogene and mostly isotropic environment
- **szemianalitic model**
- **numerical model**:
- an iterativ, matematically not egzact solution of the groundwater, solute and/or heta transport equations
- discretization in space and time, inside a given element at a given time step everything is constant





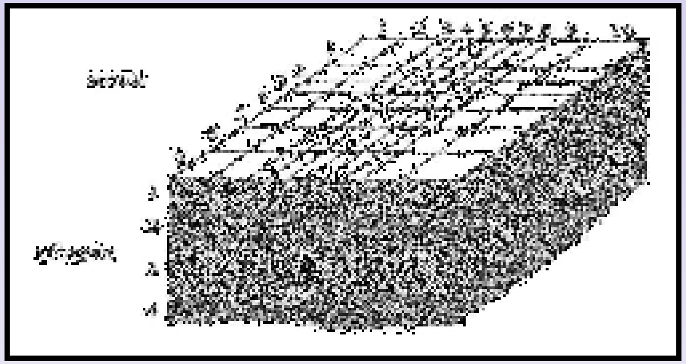
(Tari Cs., 2011)

The process of numerical modeling



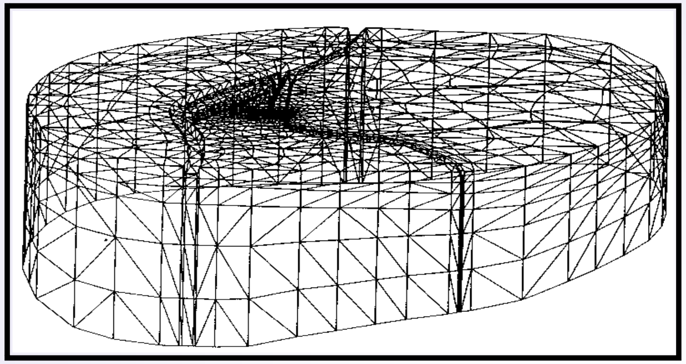
Numerical methods applied in the practice

- **Finite difference method:** the modelled domain is discretized by a rectangular grid where all the neighbouring elements contacts each other by their sides. The system is desctibed by an equation system consisting of the water budget af the elements and the unknown parameter is the average hydraulic head (GW flow potential) in the element. The equation system solved iteratively using different numeric techniques
  - advantage: Ease of use, partial results has physical meaning
  - disadvantage: restrictions in shape of elements



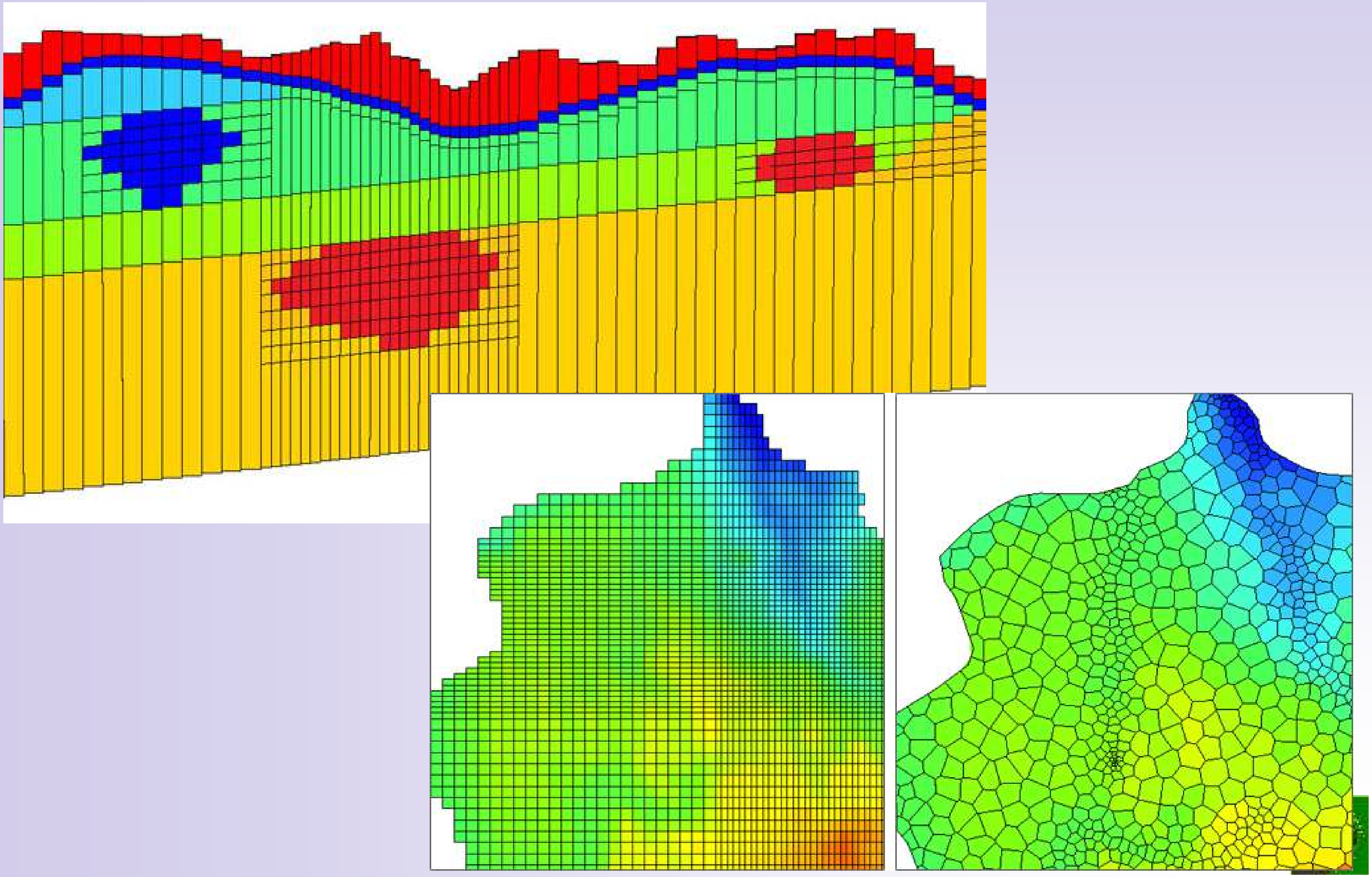
Finite difference grid of a four laayer system (CHIANG és KINZELBACH, 1999)

- **Finite elemet method:** The domain is divided into elemnts of different shape/dimension which are contacted to each other by the nodes only. Any element shape is possible until the form can be described mathematically. The resulted flow, chemical or heat potential field fits the best the values on the nodes (using local approximation). The potential distribution at the edges of neighbouring elements could be different at the different sides orof the edges . The continuity on neigbouring elemnt level is not assured: there can be a difference in in and outflowing fluxes ata side of the element.
  - advantage: any shape of elements
  - disadvantage: black box usage, high mathematics needed, no meaning of results during the run, only end results are to be evaluated

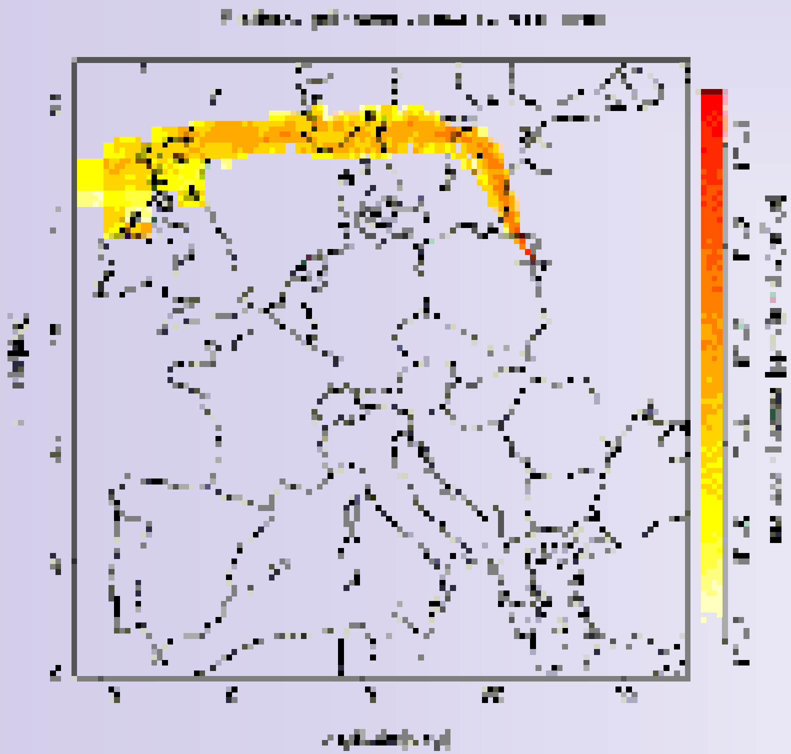


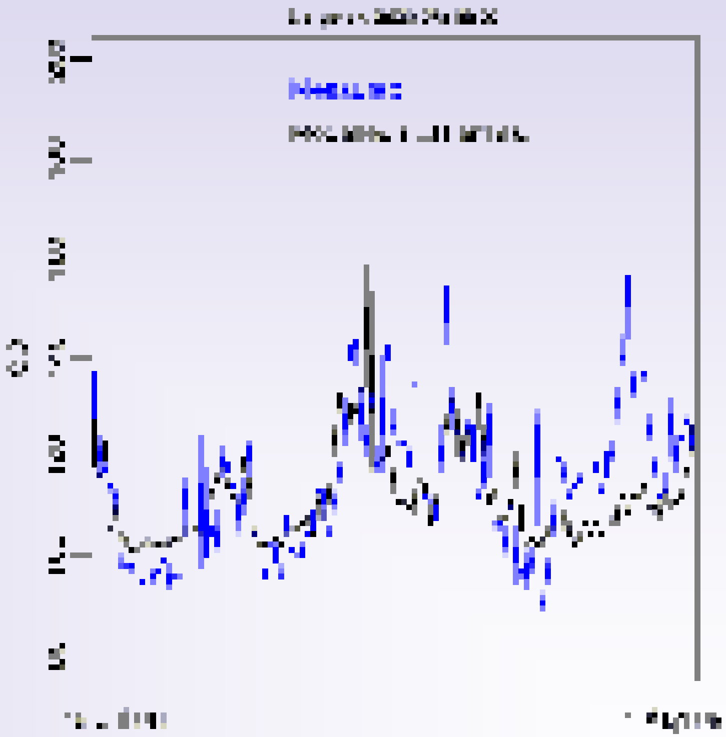
Finite element grid of a contaminant transport problem (VOGT, 1993.)

Finite volume – USG-unstructured grid



Handling suddenness: stochastic and deterministic modeling





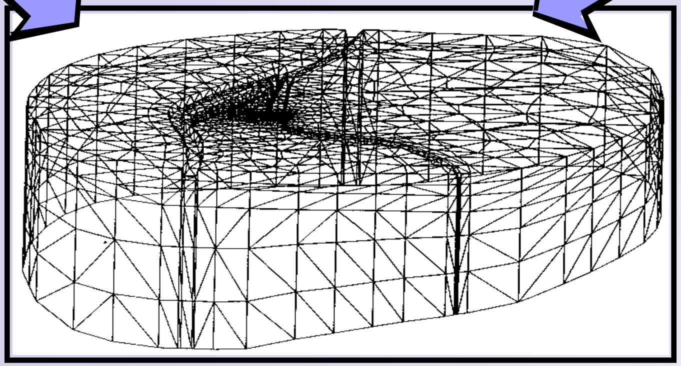
STILT, the **S**tochastic **T**ime-**I**nverted **L**agrangian **T**ransport model

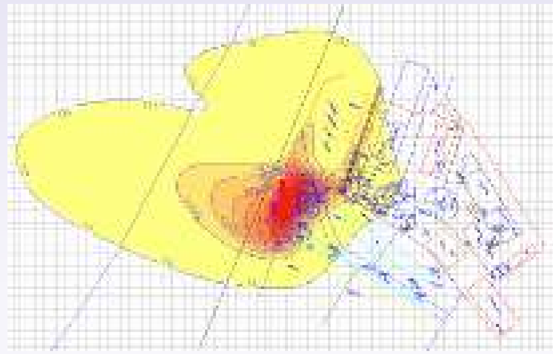
A deterministic modeling

K, n, S_s, S_y, K_d, α, λ

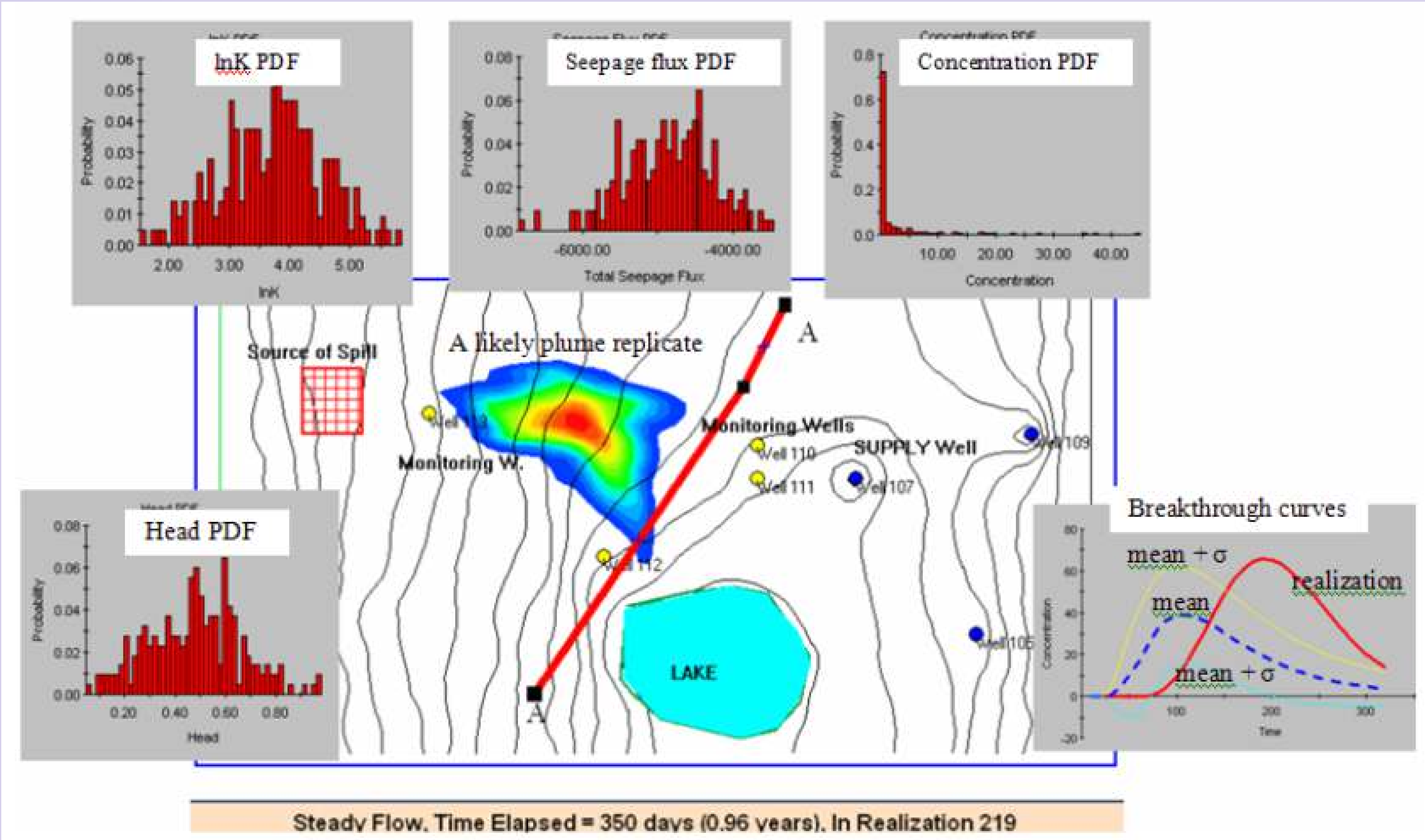
$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial h}{\partial z}\right)=\rho S_s \frac{\partial h}{\partial t}$$

$$\frac{dM}{ndVdt}=D^*_{xx}\frac{\partial^2 C}{\partial x^2}+D^*_{xy}\frac{\partial^2 C}{\partial x \partial y}+D^*_{xz}\frac{\partial^2 C}{\partial x \partial z}+D^*_{yx}\frac{\partial^2 C}{\partial y \partial x}+D^*_{yy}\frac{\partial^2 C}{\partial y^2}+D^*_{yz}\frac{\partial^2 C}{\partial y \partial z}+ D^*_{zx}\frac{\partial^2 C}{\partial z \partial x}+D^*_{zy}\frac{\partial^2 C}{\partial z \partial y}+D^*_{zz}\frac{\partial^2 C}{\partial z^2}-\frac{\partial}{\partial x}\left(\frac{v_x C}{n}\right)-\frac{\partial}{\partial y}\left(\frac{v_y C}{n}\right)-\frac{\partial}{\partial z}\left(\frac{v_z C}{n}\right)-\frac{\partial}{\partial t}\left(\frac{\rho_b K_d C}{n}\right)-\lambda\left(C+\frac{\rho_b K_d C}{n}\right)$$



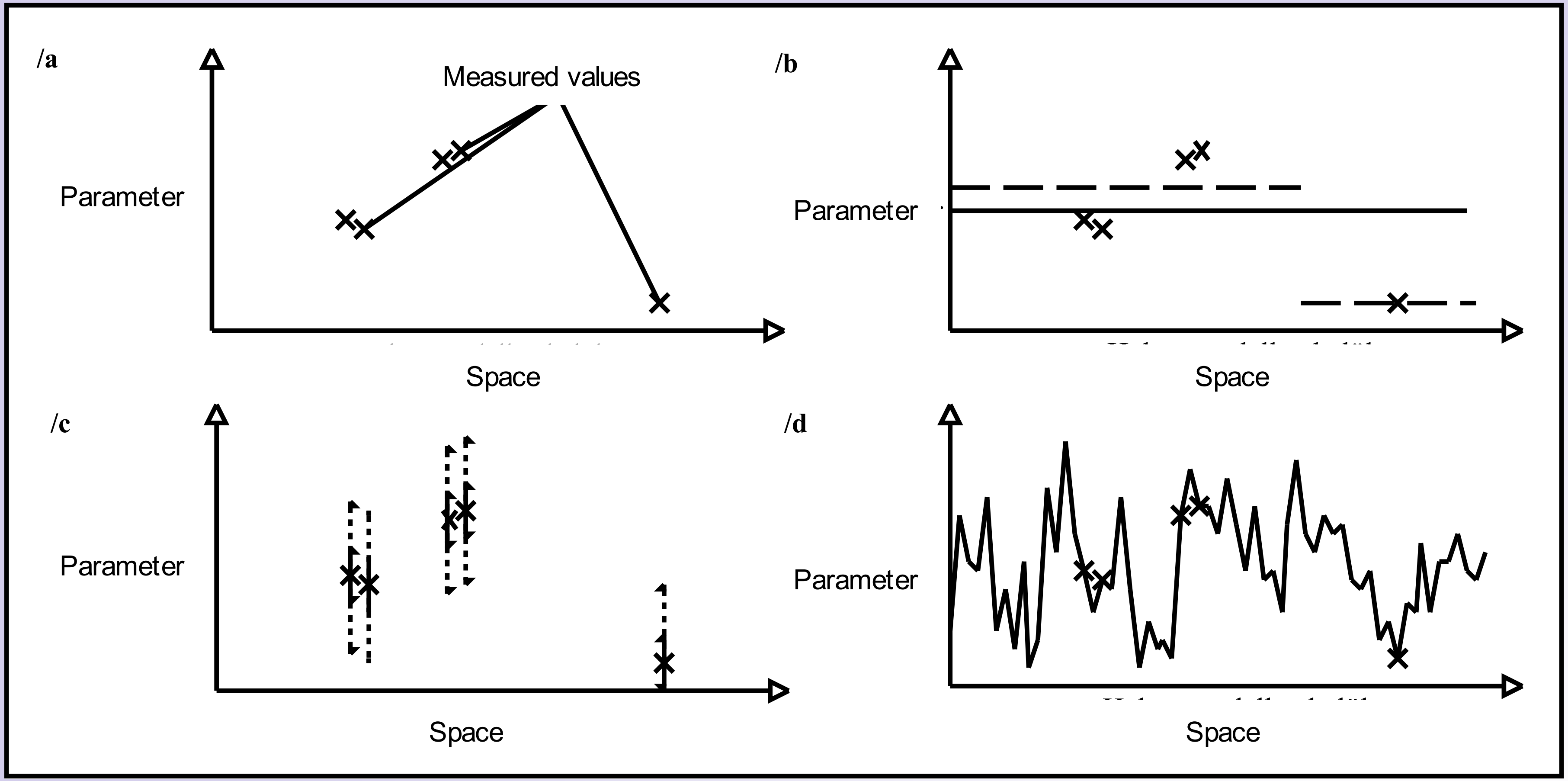


Stochastic modeling

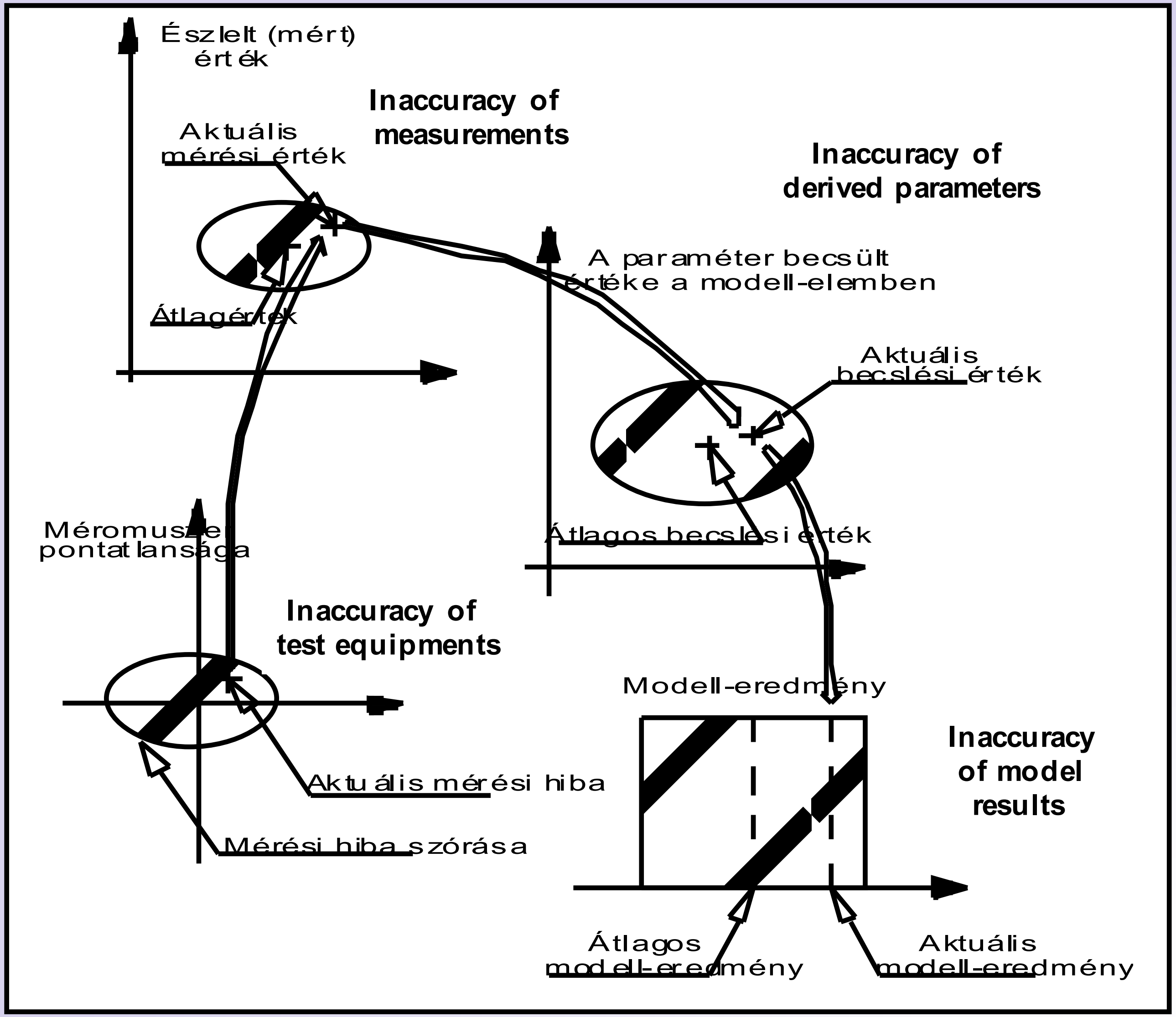


Shu-Guang Li, Hua-Sheng Liao, Qun Liu, 2005
National Science Foundation

Origin of parametric errors

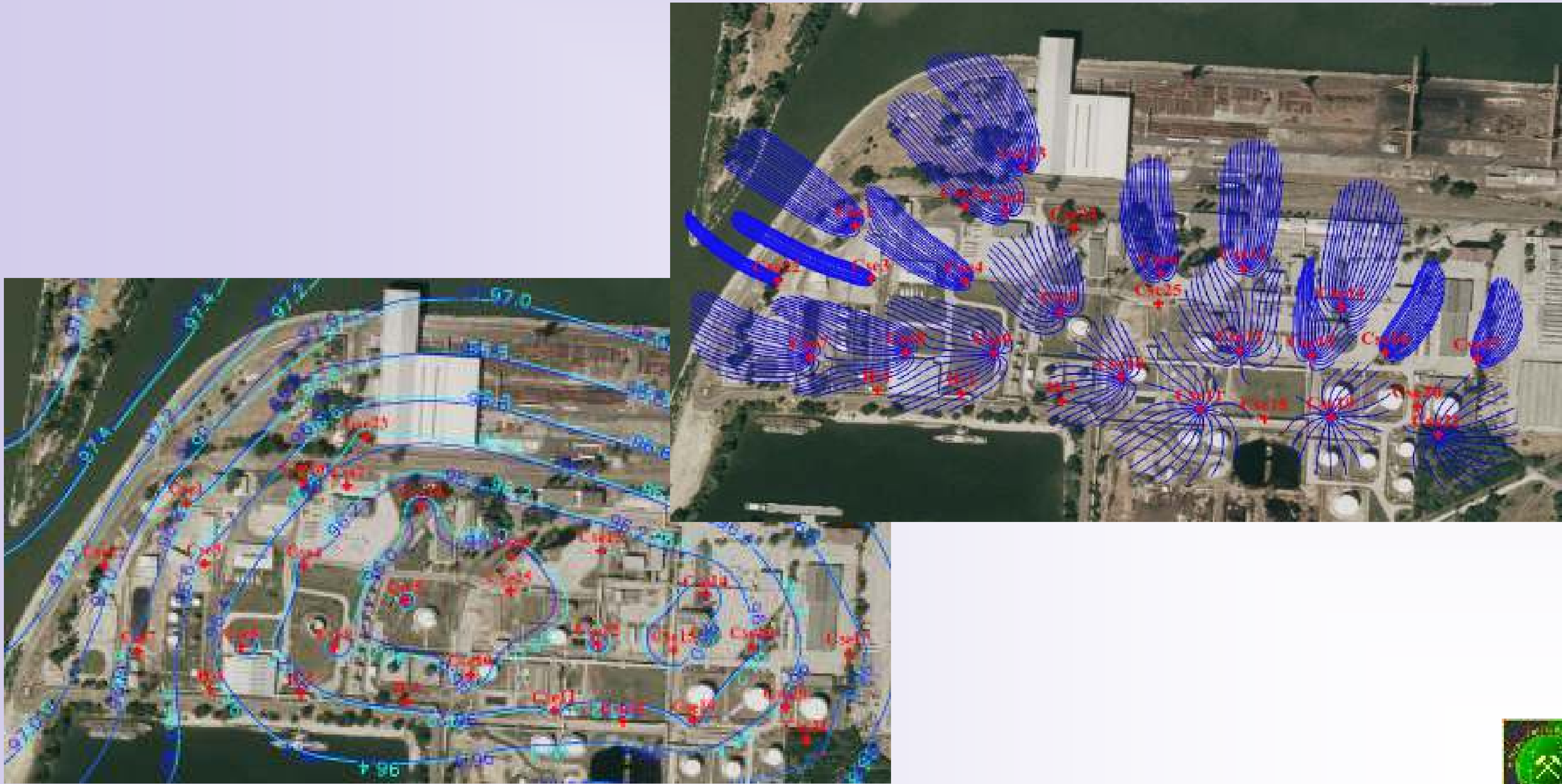


Inheritance of errors



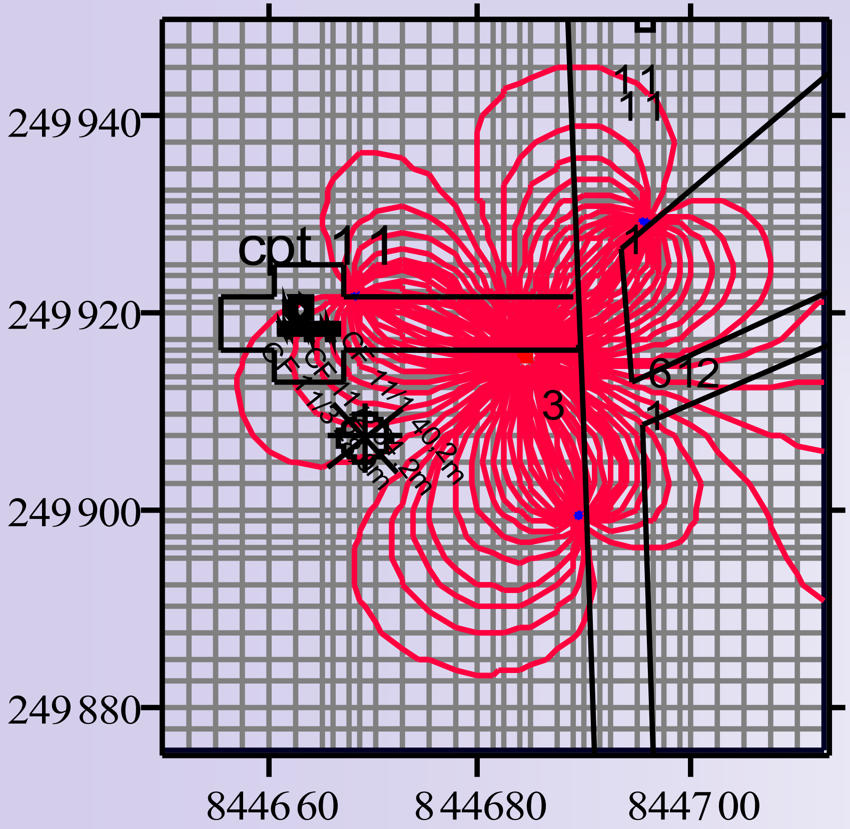
GW Flow models

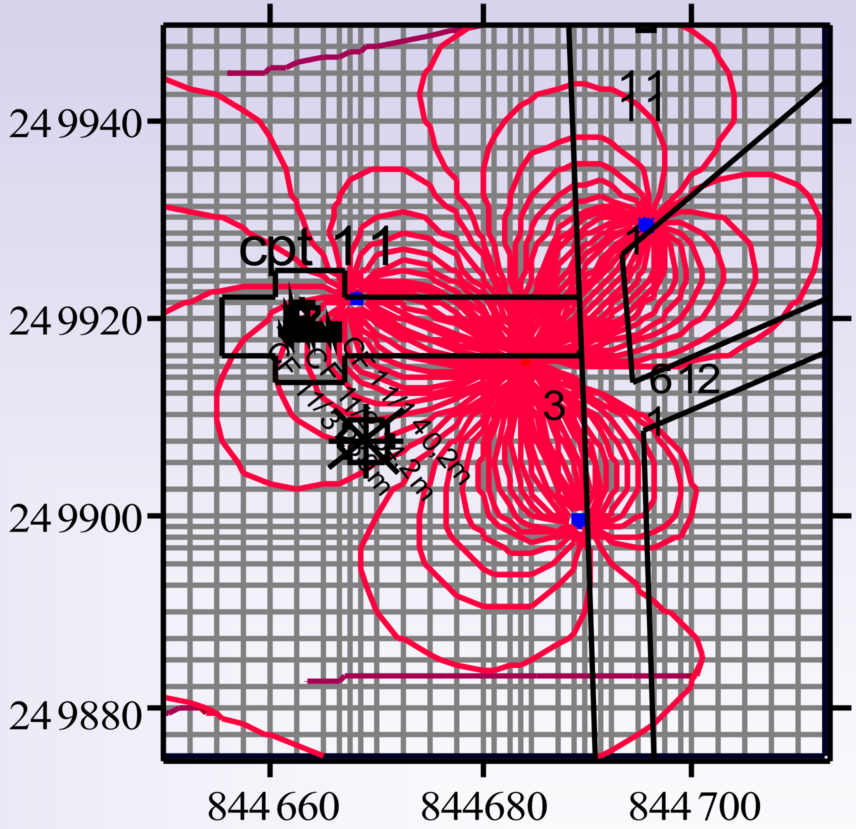
- Calculation of seepage direction and velocities
- transit time calculation



GW Flow models #2

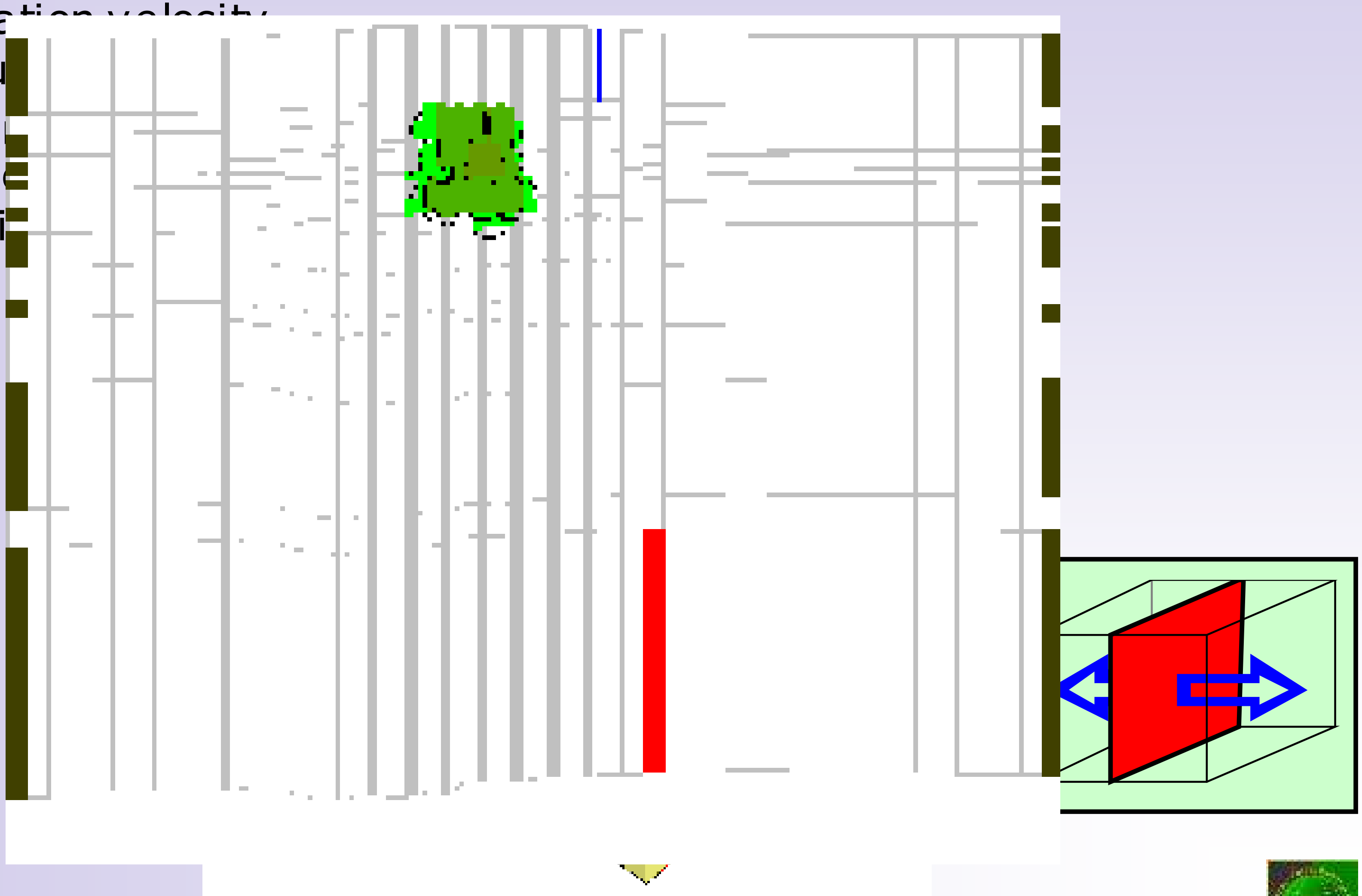
Simulation of flow system of an injection and three production wells



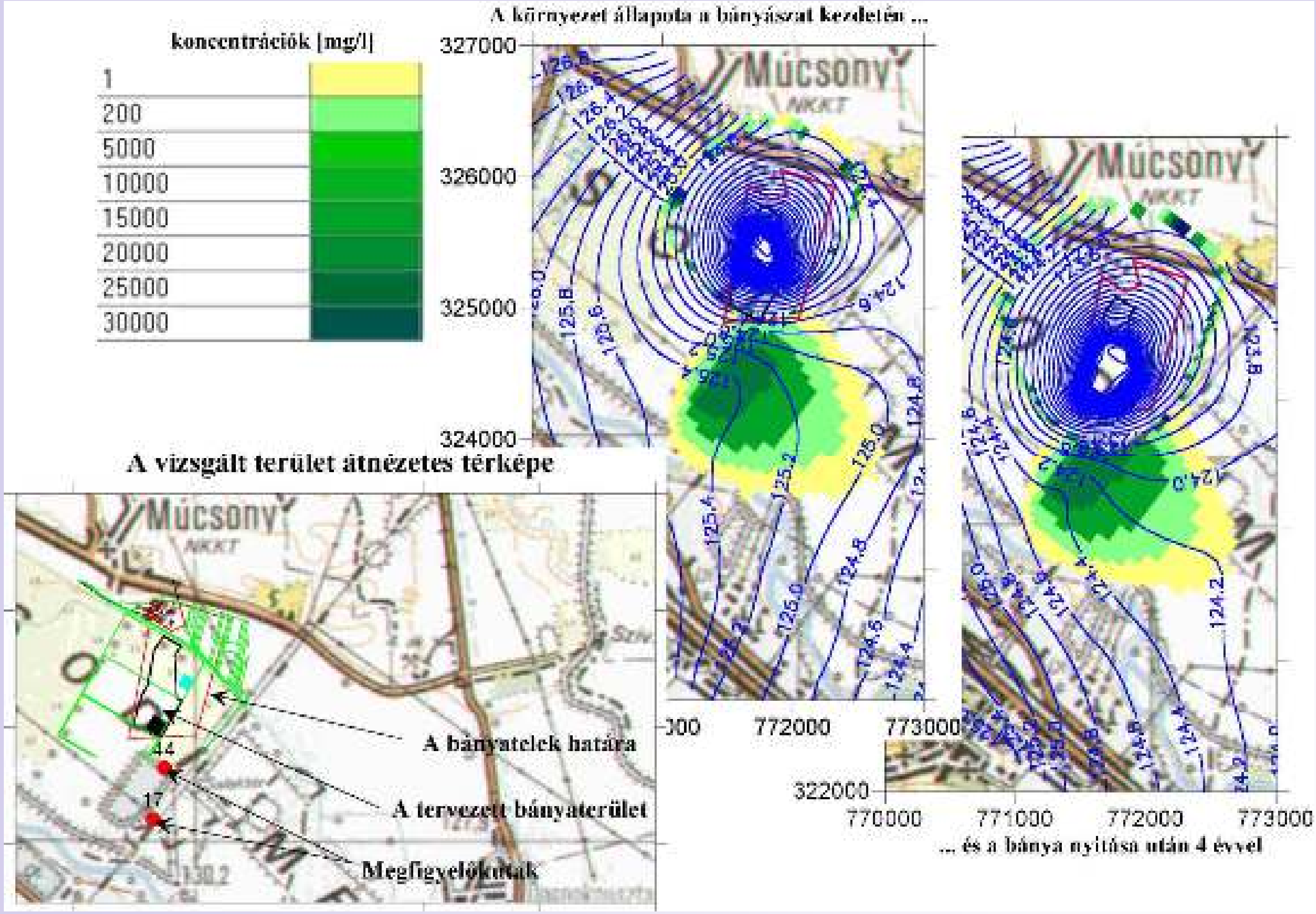


Contaminant transport models

- Migration velocity calcu
- Dete conc distri

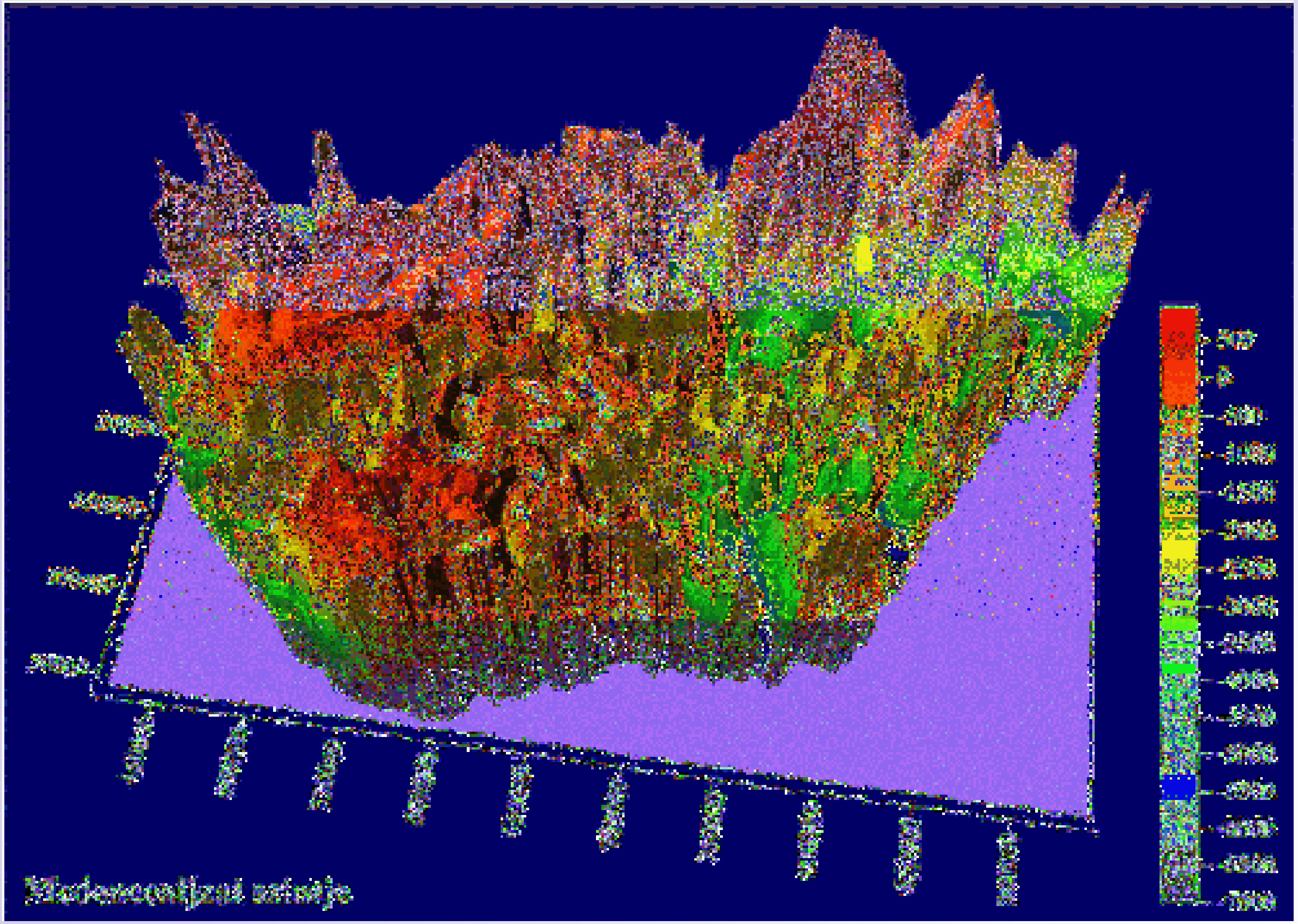


Contaminant transport models #2

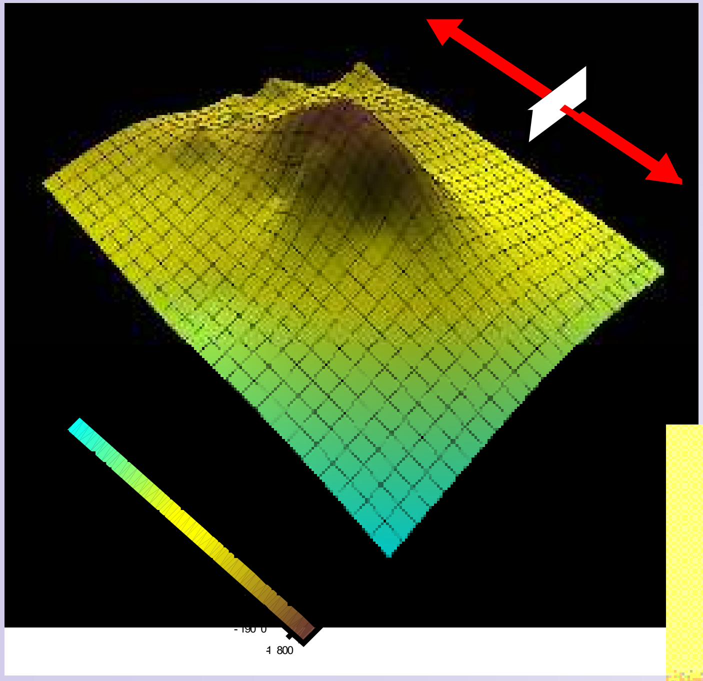


Heat transport models

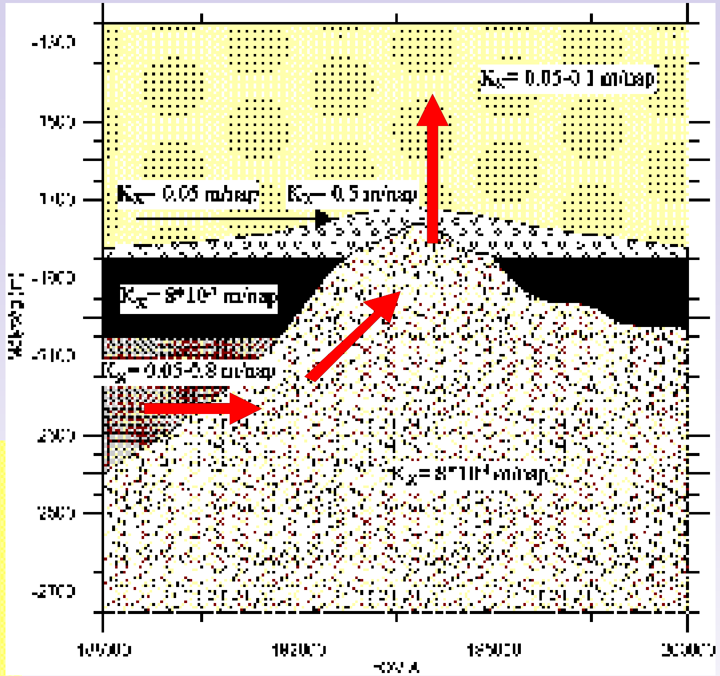
- Coupled calculations of GW flow and heat transport
- Determination of temperature distribution



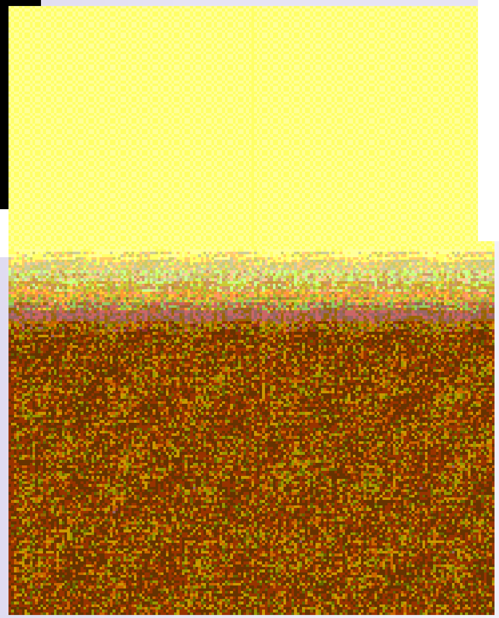
Heat transport models #2



3D surface of the dome



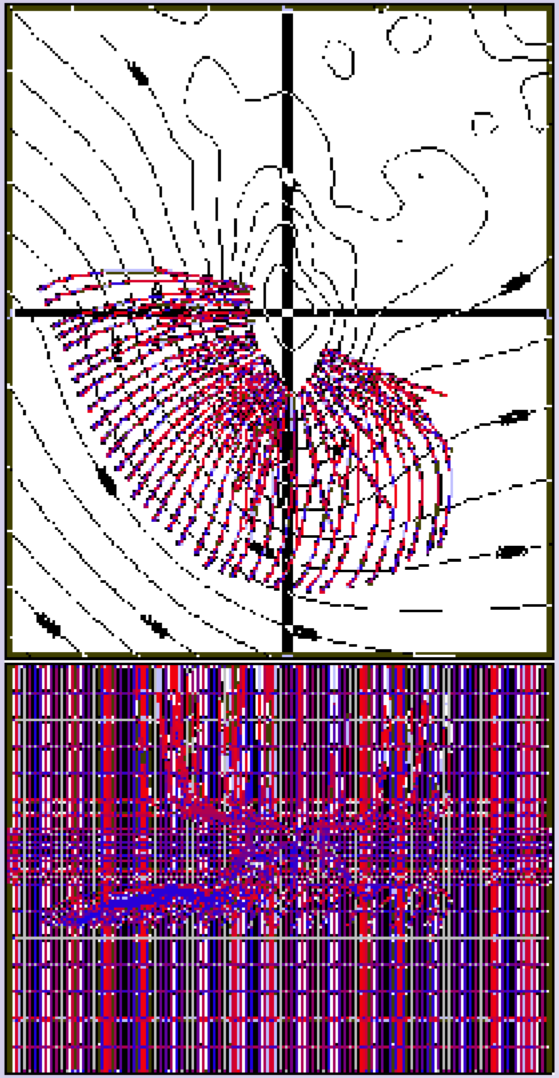
Simplified hydrogeological model



Calculated flow potential field
E-W sections

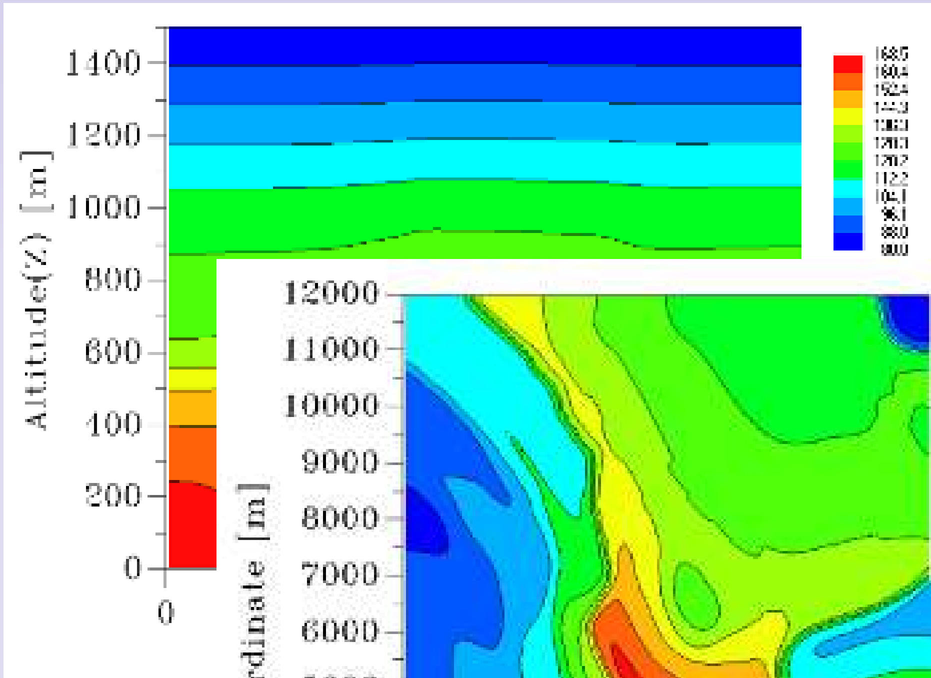
Vass I. – M. Tóth T.

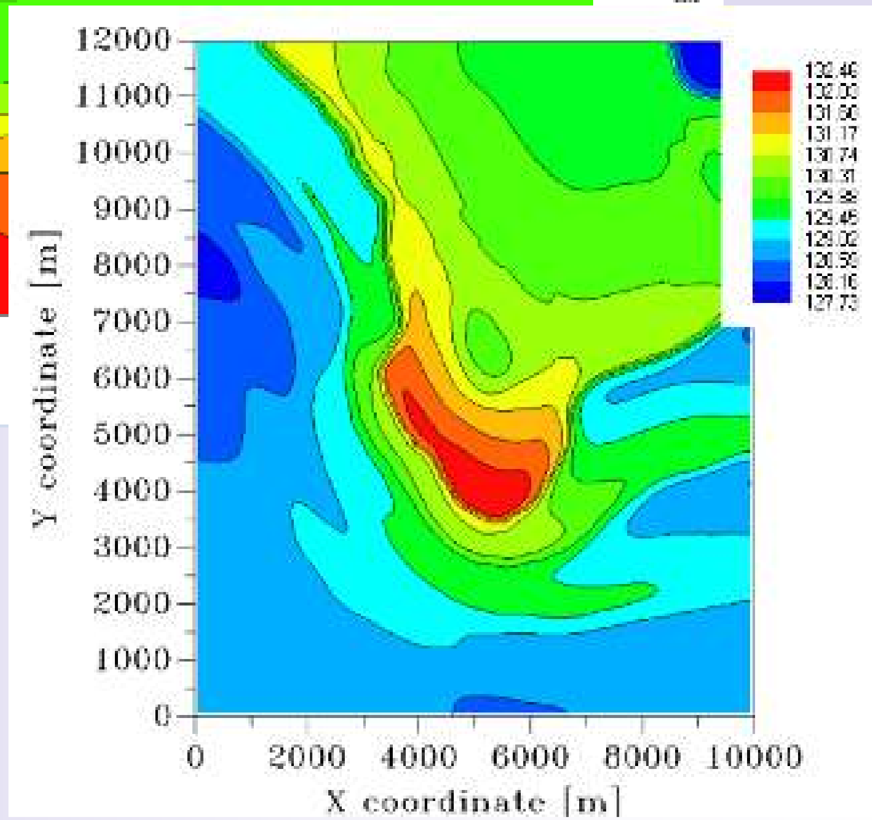
Heat transport models #3



The dome acting as a chimney

T-section





T-anomaly map

Application fields of numerical models? What we can do…

- to determine the validity ranges of legislative laws (governing equations)
- to control measurement results
- to compare different cases
- to perform
  - data sensitivity analysis
  - scernario analysis,
  - best-case worst case analysis
- etc.

What we can not do … (theoretically)

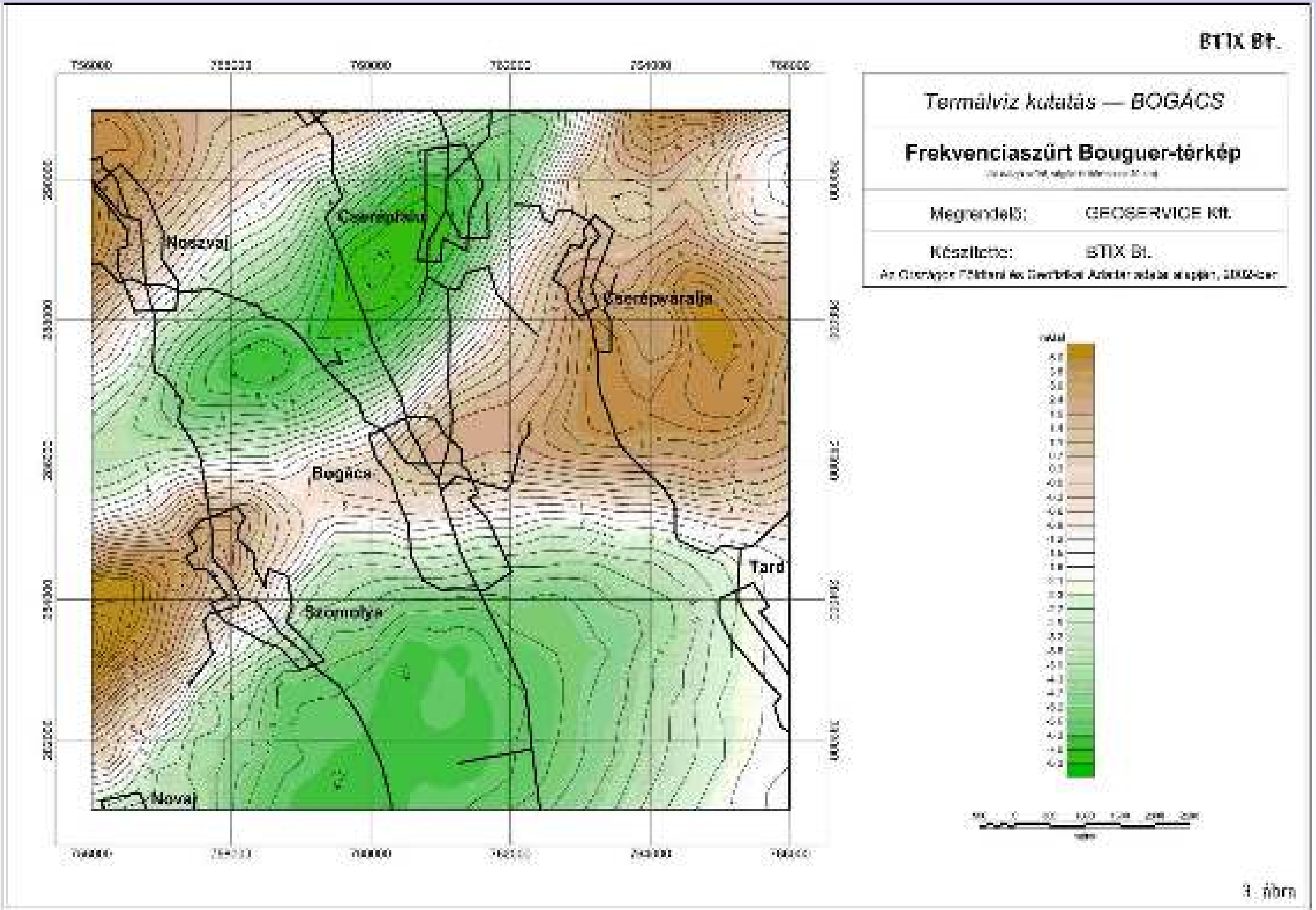
- forecasting with high accuracy
- forecasting small changes in pressure, concentration or temperature (for ex. determination of pollution above a limit value, etc)
- long term forecasting
- etc.

... but we use them for this as well having no better solution!

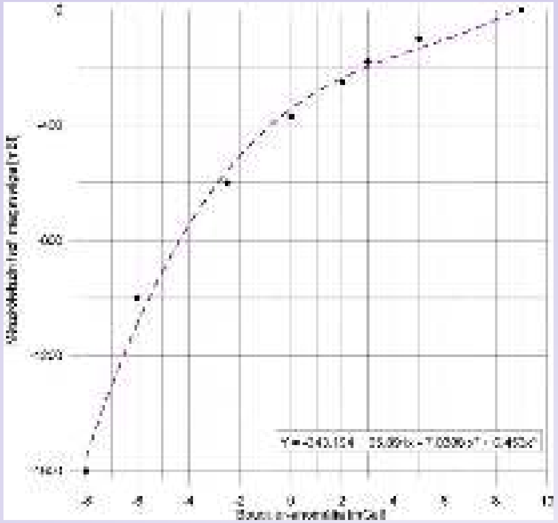
Problems

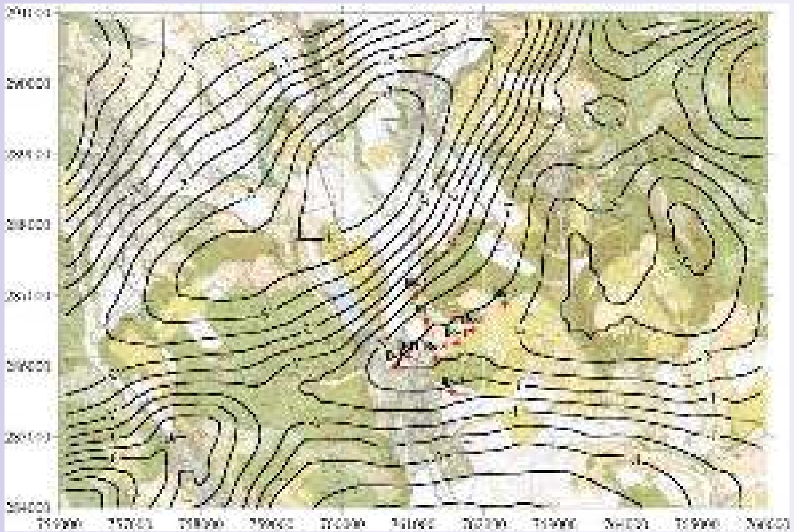
- Lack of geological, hydrogeological information of reservoirs
- Inhomogeneity of geological formations
- Handling random behavior
- Handling and simulating real world dynamics (in some case the reality is too quick, in another case it is too slow...
- Unknown relevant mechanisms of the real systems or misunderstanding the real world processes
- Inaccuracy of governing equations
- Numerical errors
- and so on...

Use of geophysical gravity measurements to overcame the lack of geological information to determine WHPA of gothermal wells

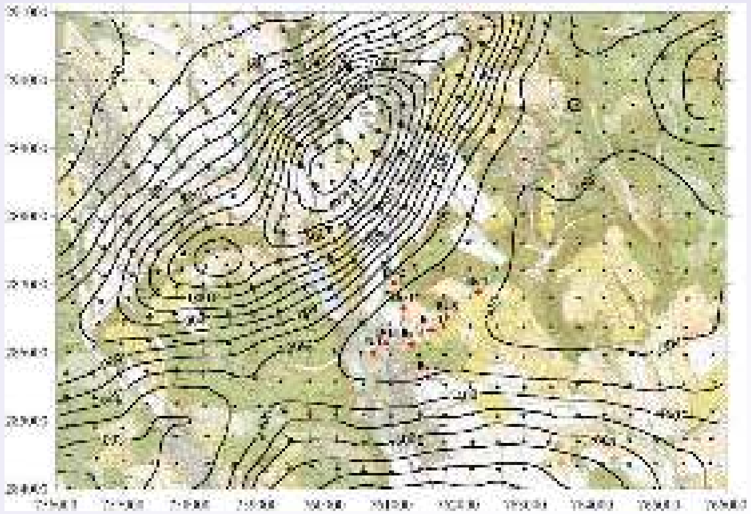


Use of gravity measurements at Bogács thermal well modeling



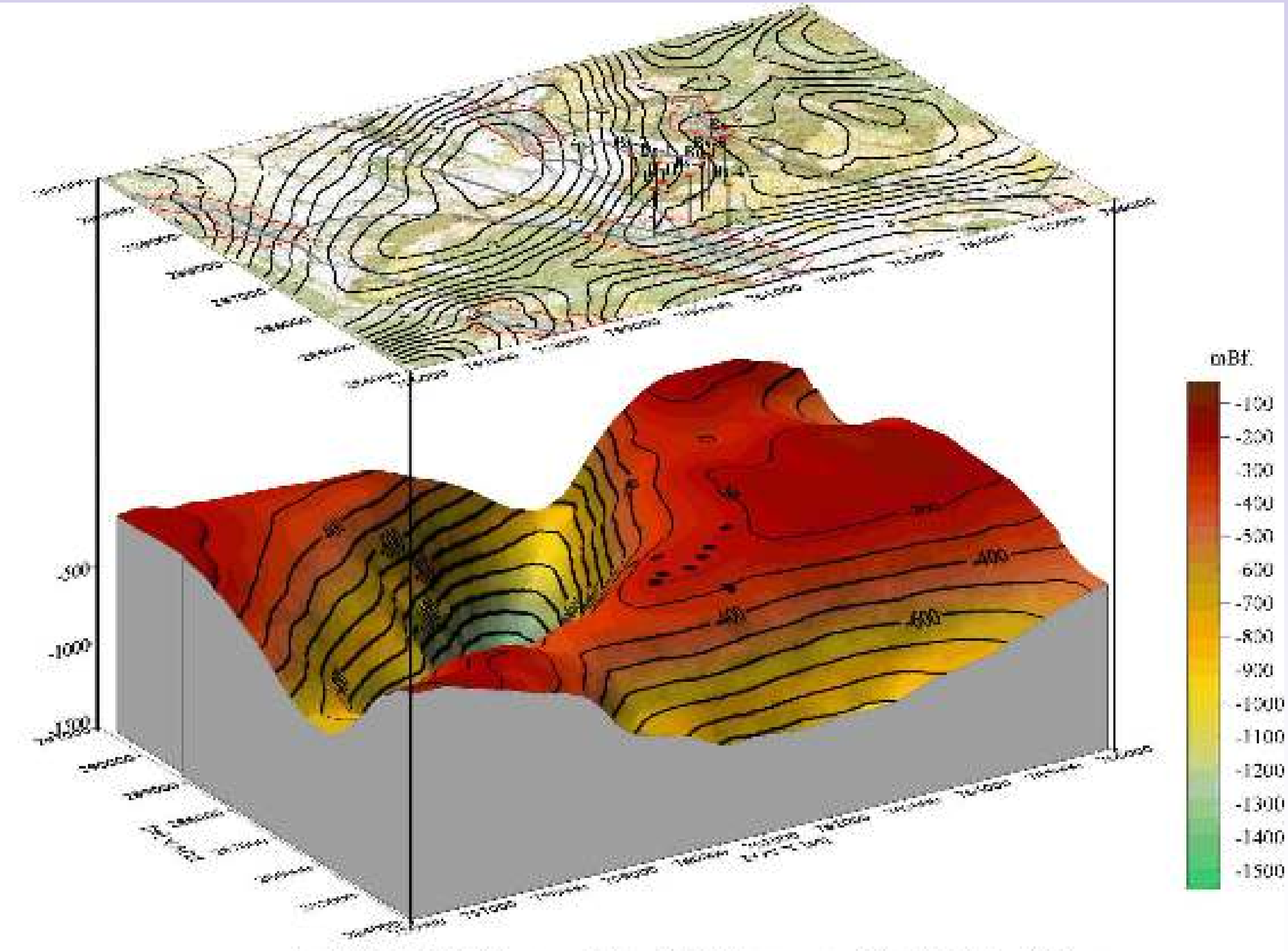


Bouguer anomaly map



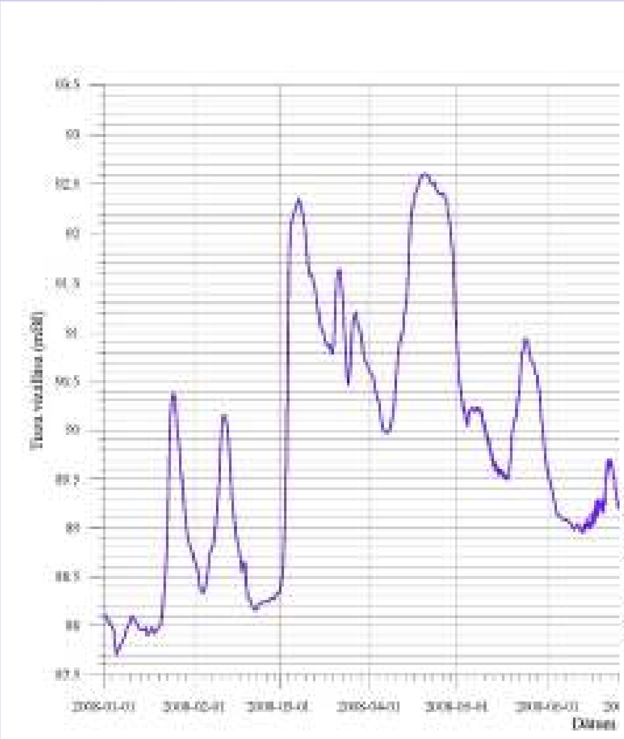
Hipothetic depth distribution

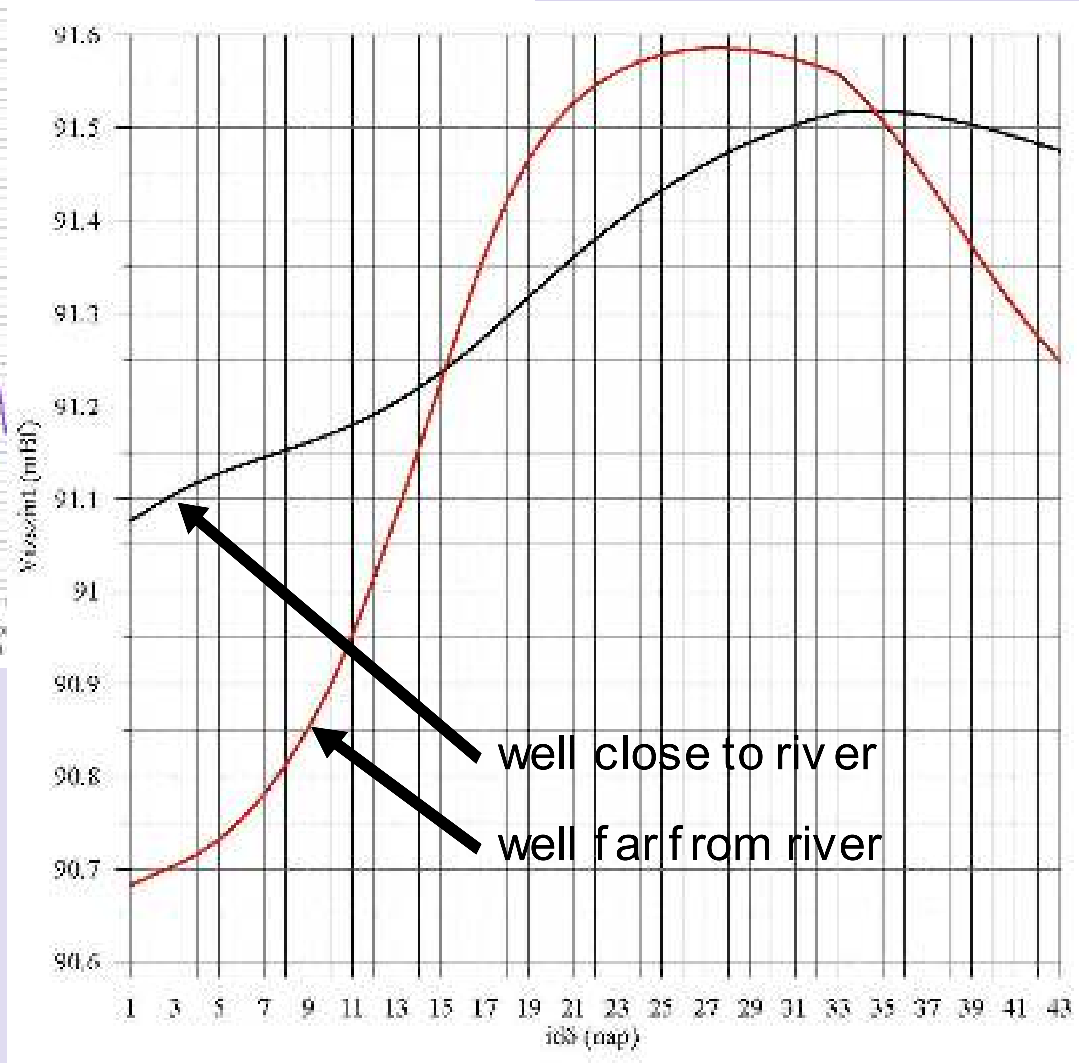
3D interpretation of gravity map data



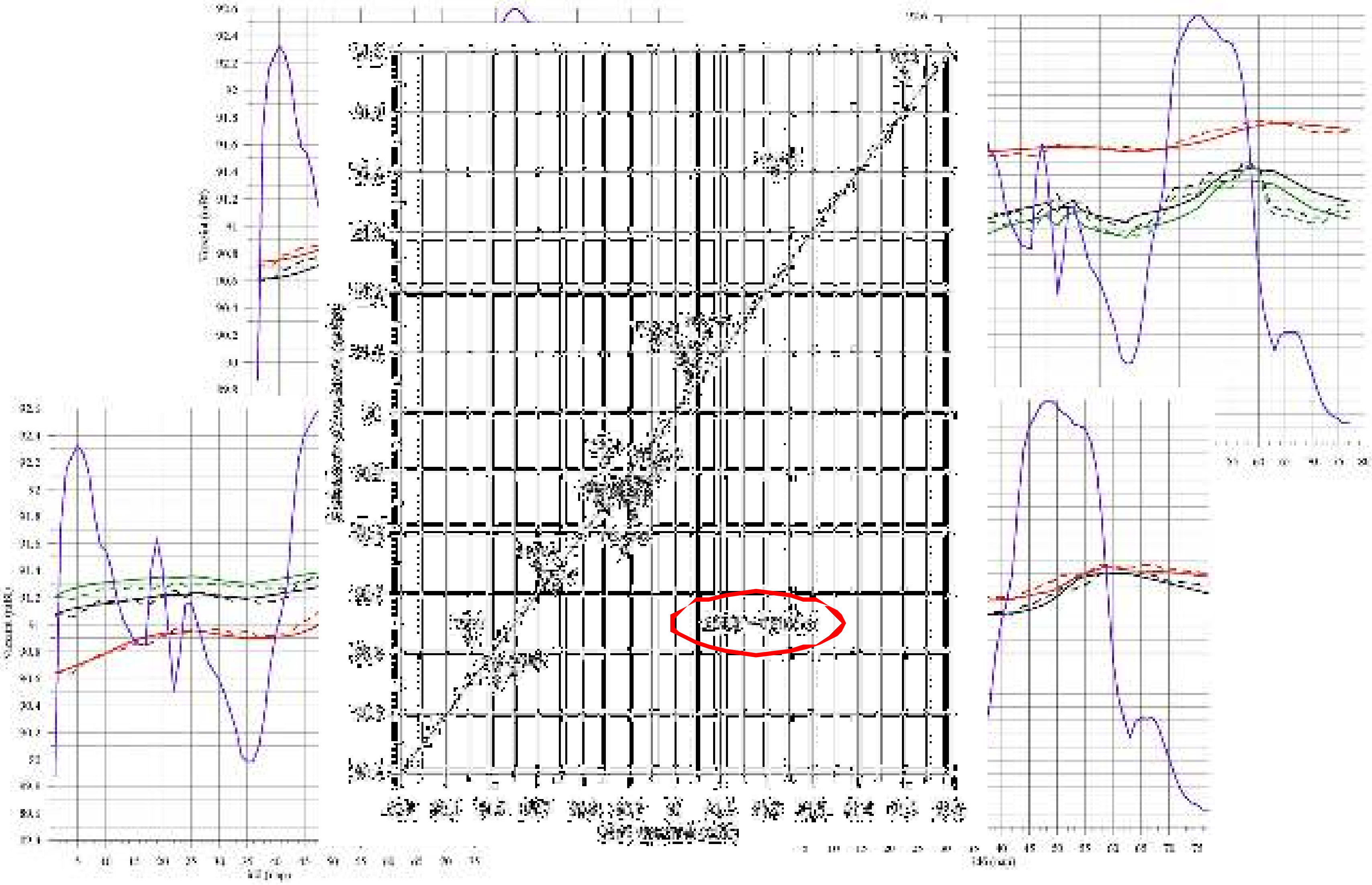
A mészkőfelszín [mBf., 2x magassági torzítás] a Bouguer-anomáliák [mGal] összefüggése

Effect of riverhead changes

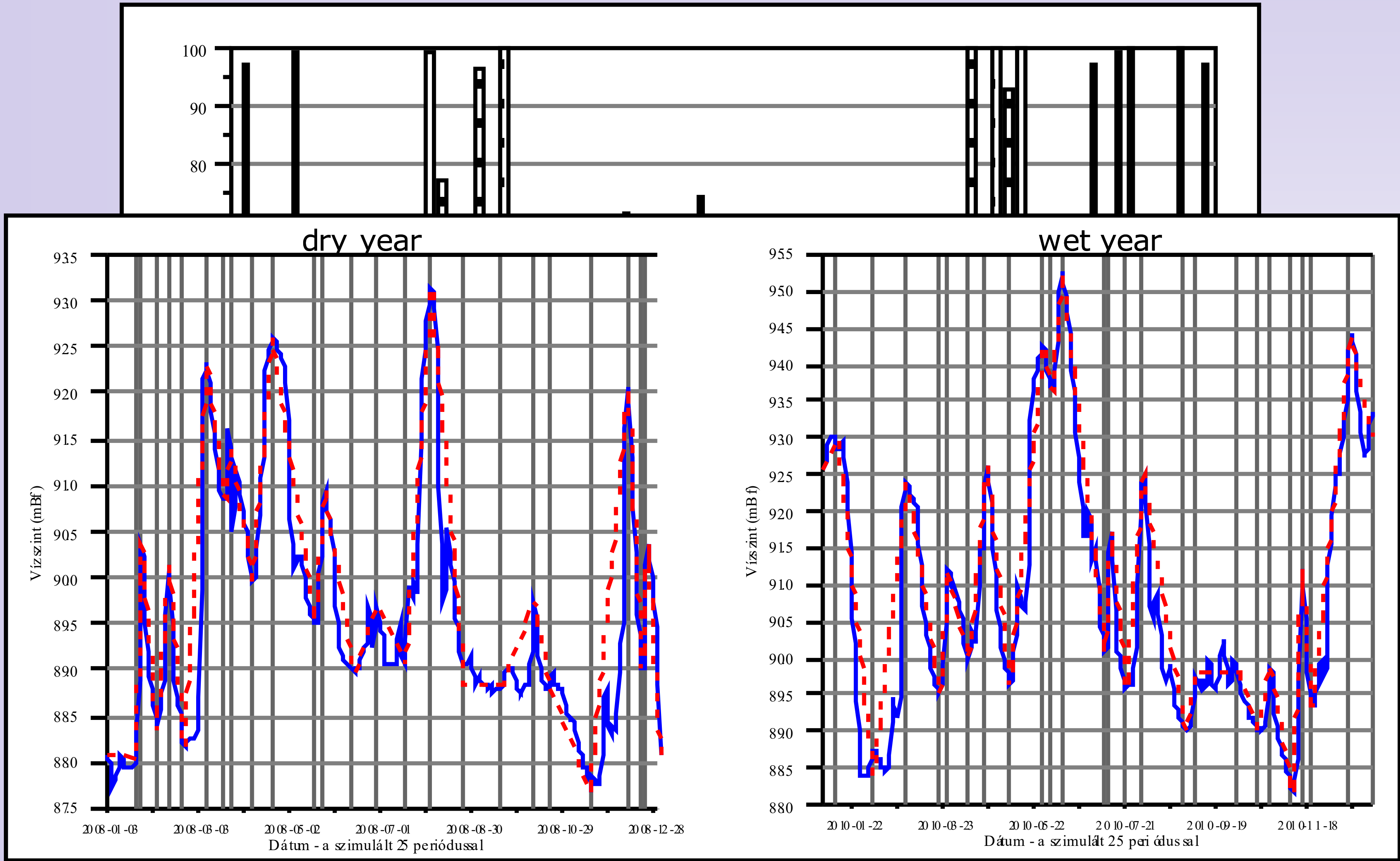


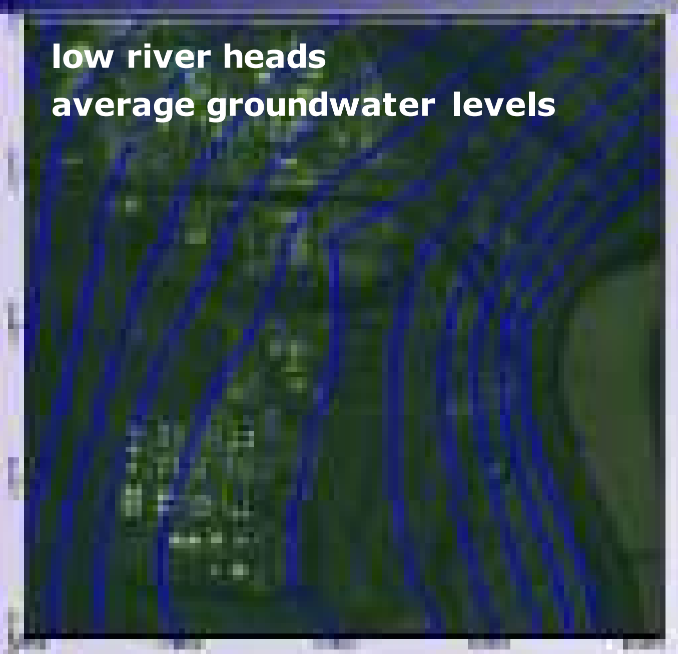


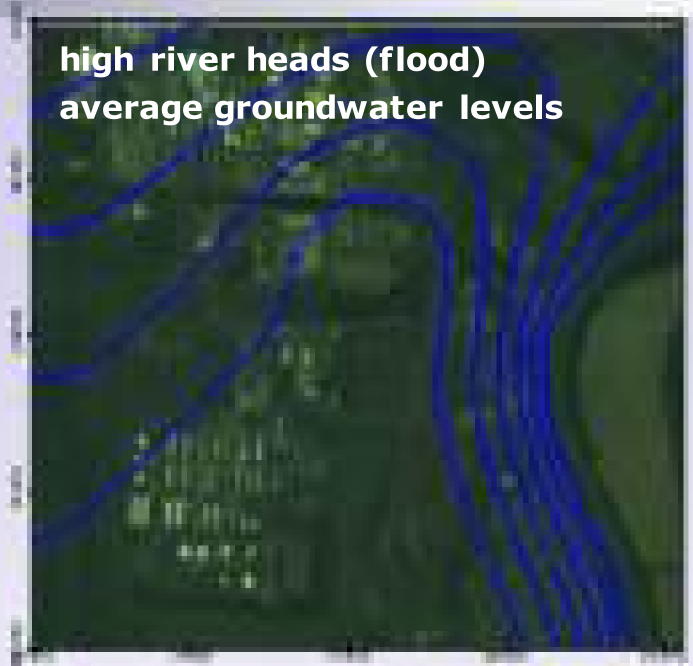
Transient calibration



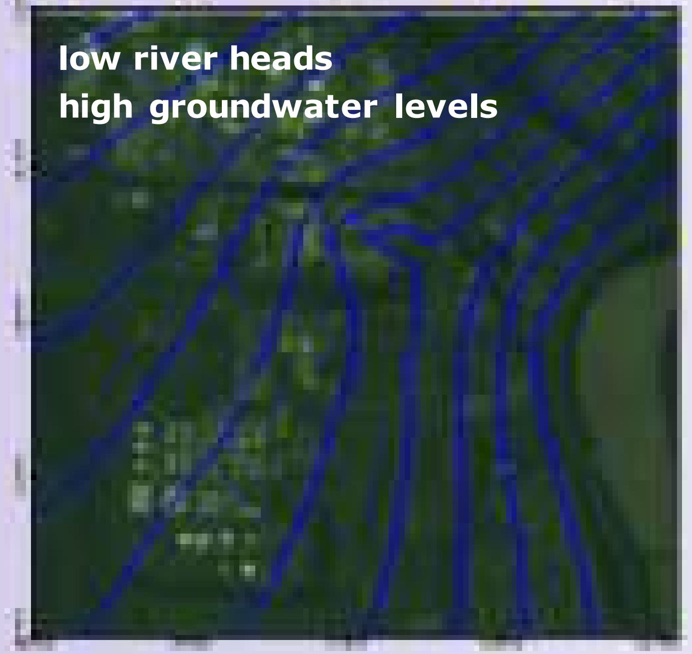
Modification of hydrodynamics of the model at dry and wet conditions

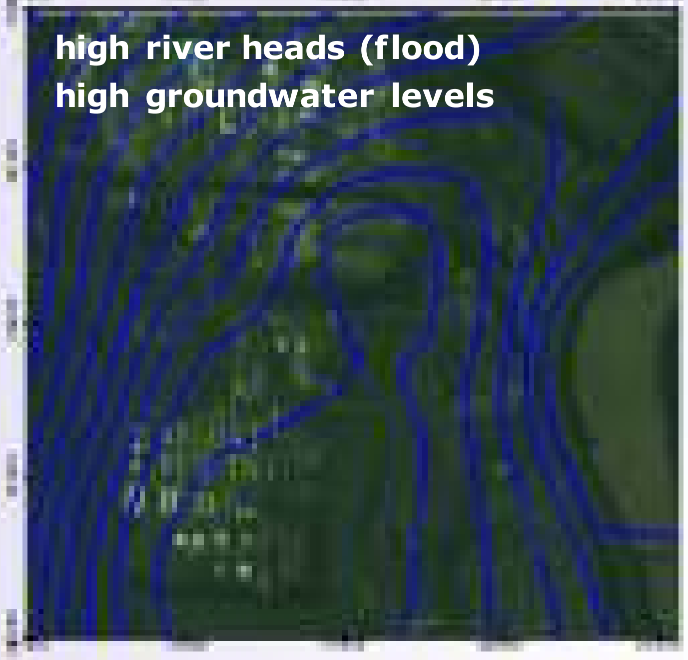






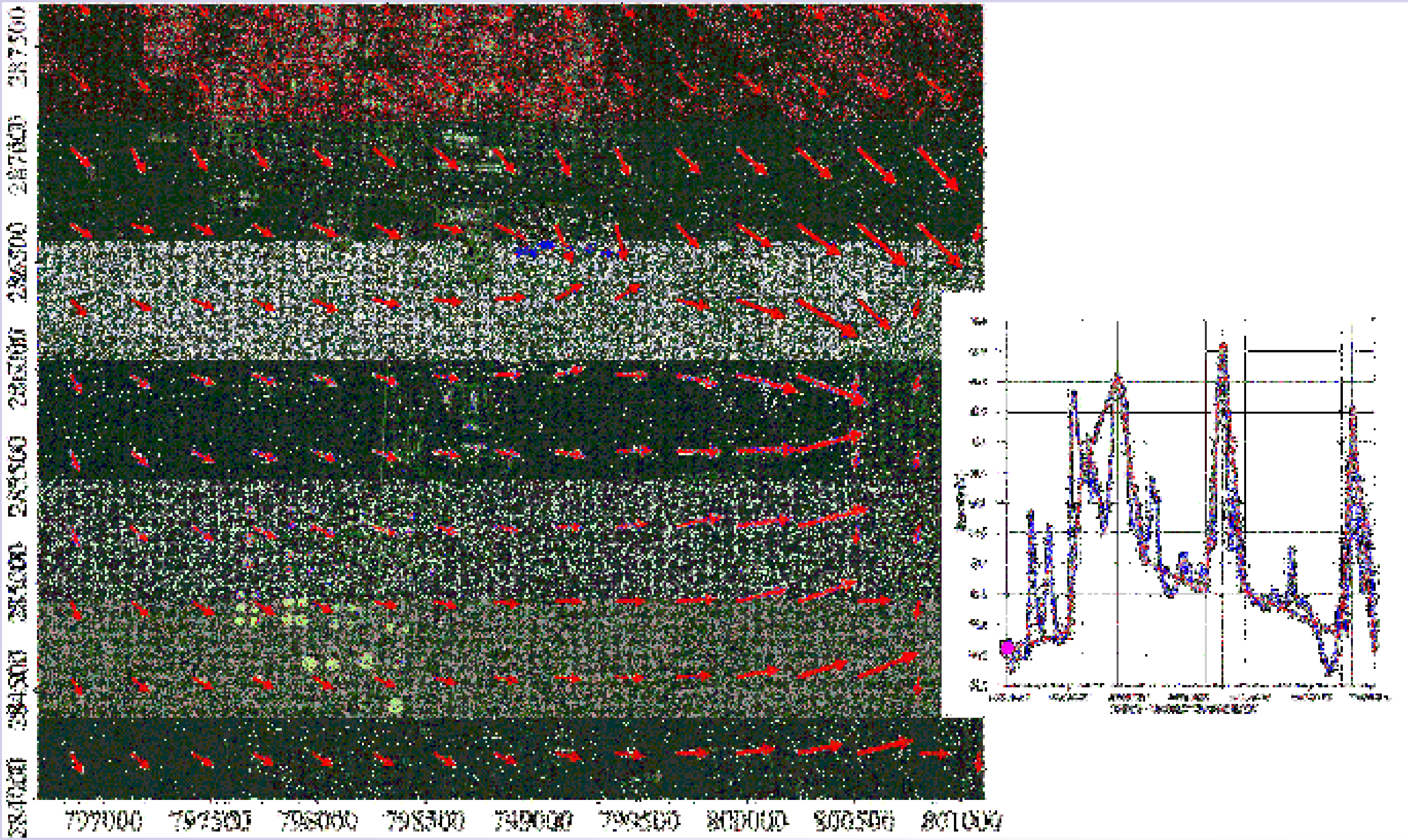
dry year



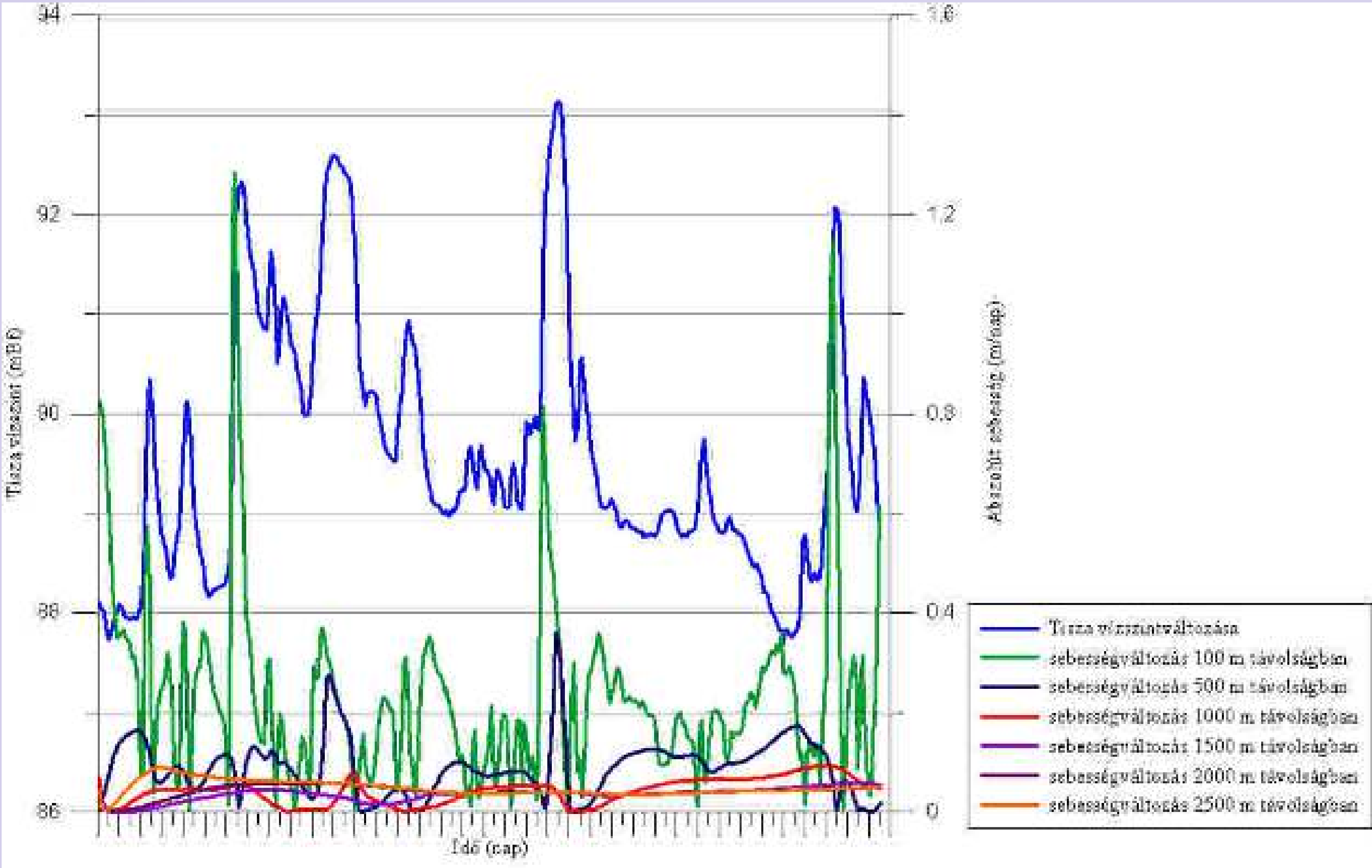


wet year

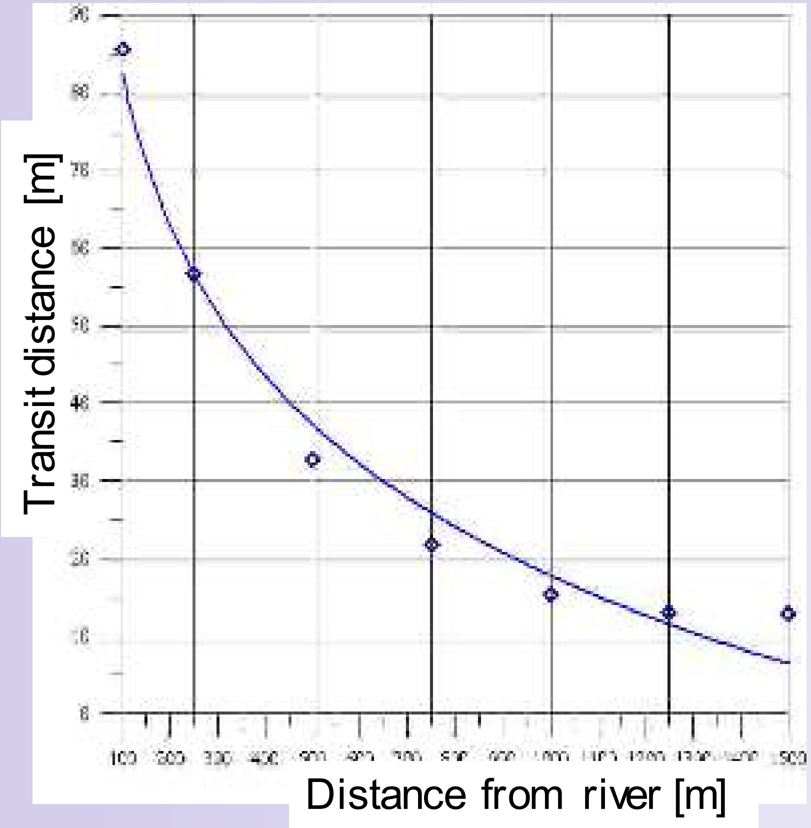
Dynamics of GW – SW intreractions

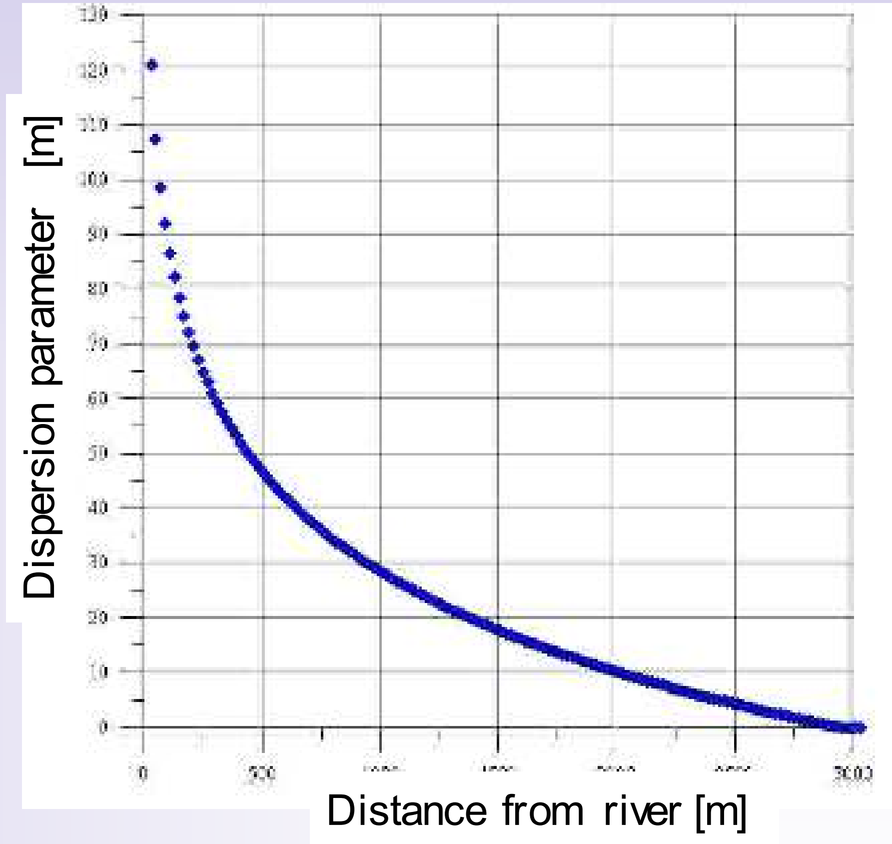


Seepage velocity characteristics

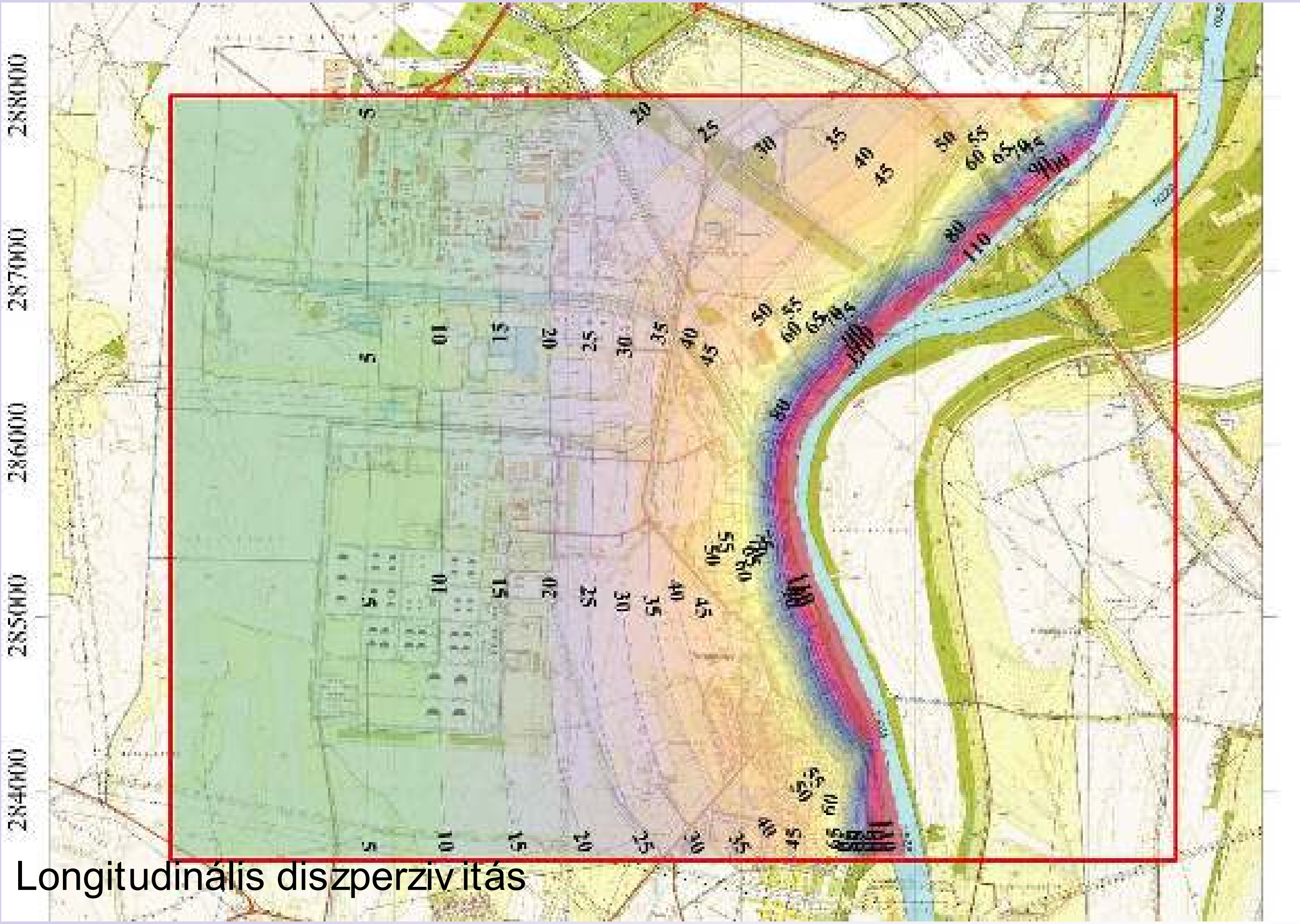


Dispersion and transit distance vs. distance from river



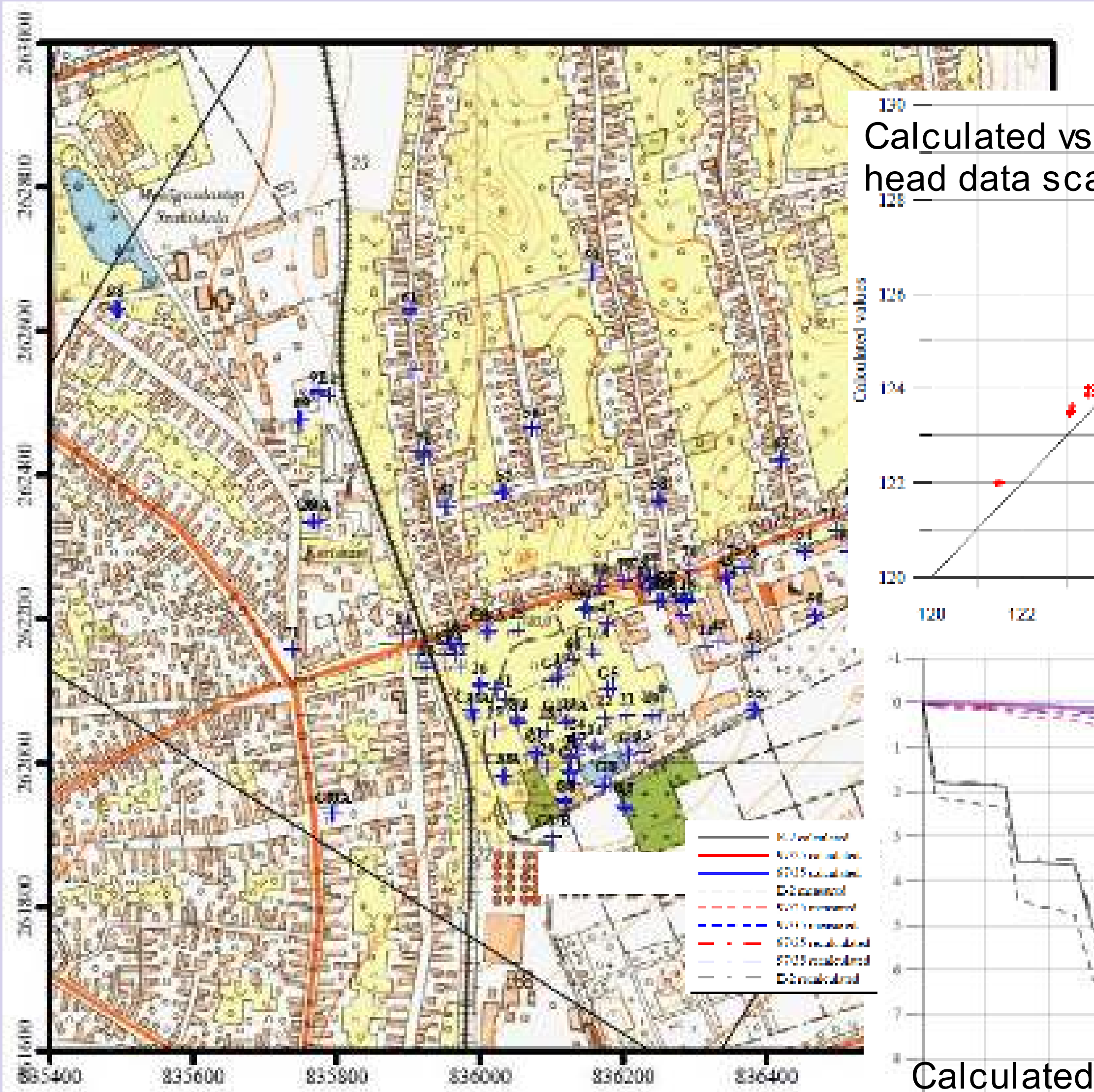


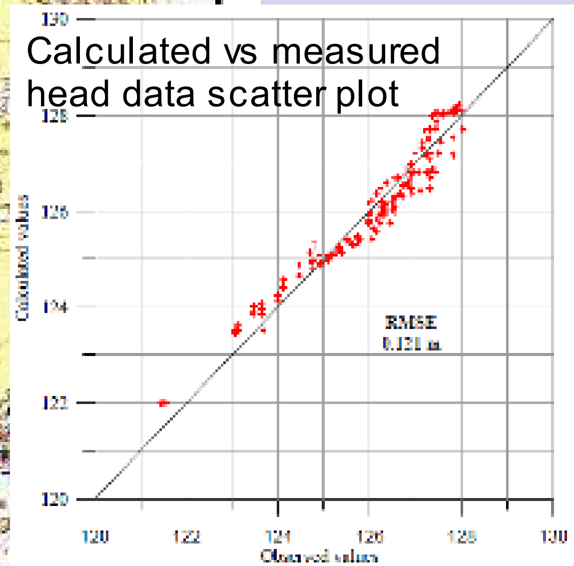
Hydrodinamic dispersivity [m] distribution



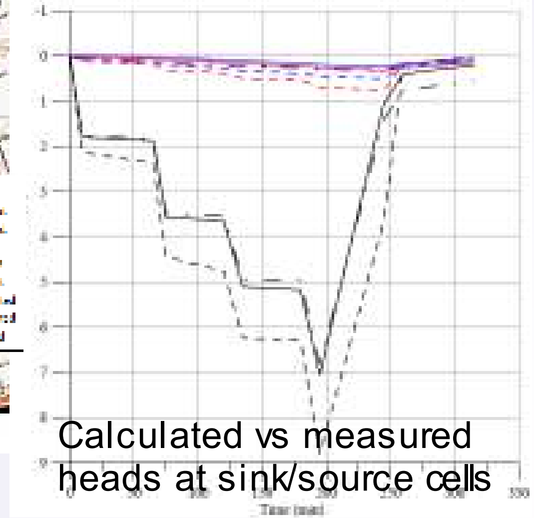
Longitudinális diszperziv itás

Use of calibration points





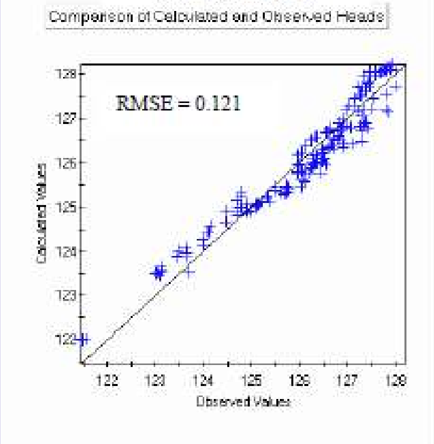
Calculated vs measured head data scatter plot



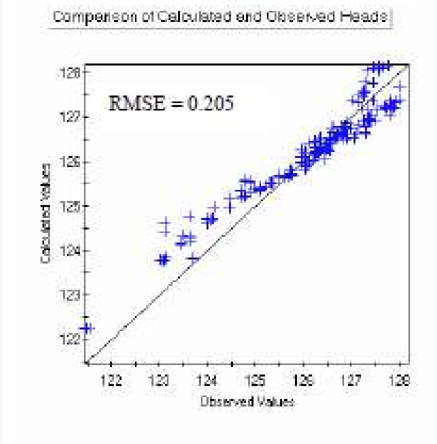
Calculated vs measured heads at sink/source cells

Input data sensitivity analysis

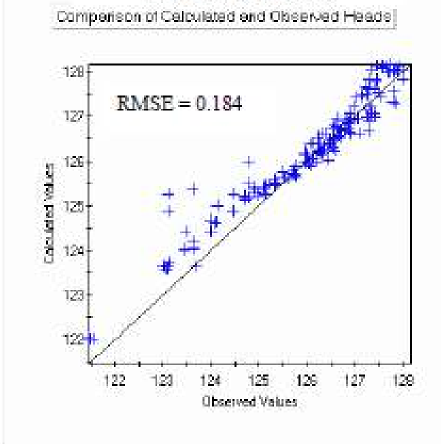
Kalibrált adatrendszer
Calibrated dataset



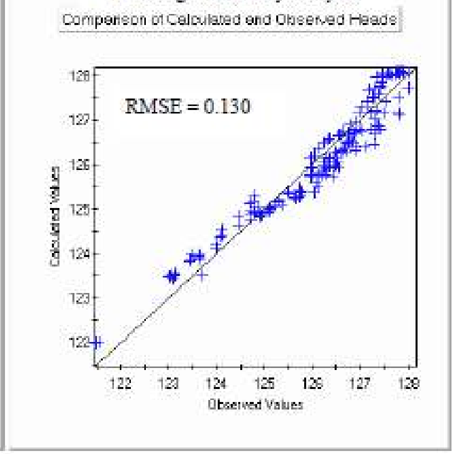
Vízszintes szivárgási tényező vizsgálata
Horizontal hydraulic conductivity analysis



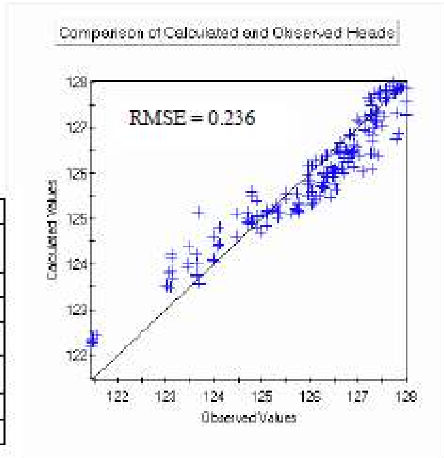
Vízszintes és függőleges szivárgási tényező együttes vizsgálata
Horizontal and vertical hydraulic conductivity analysis

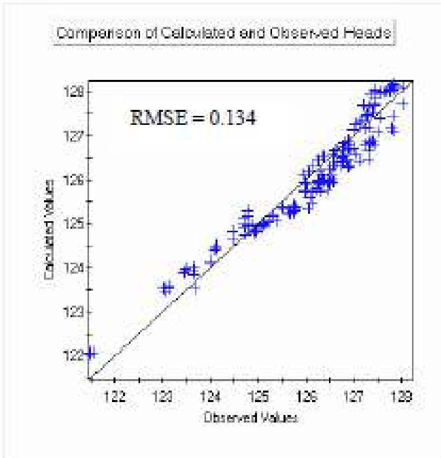


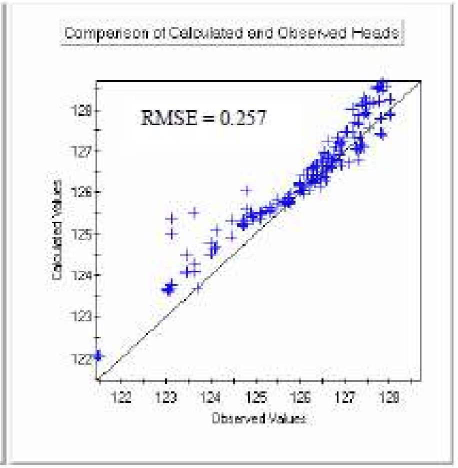
Beszivárgás vizsgálata
Recharge sensitivity analysis



Paraméter	Nagyságrend	RMS hiba
Kalibrált adatrendszer	0	0.121
Horizontális szivárgási tényező	-1 (0.1x)	0.205
	+1 (10x)	0.236
Horizontális és vertikális szivárgási tényező	-1 (0.1x)	0.184
	+1 (10x)	0.134
Beszivárgás	-1 (0.1x)	0.130
	+1 (10x)	0.257







Whether modeling or real life, never give up easily!”

(Wen-Hsing Chiang， creator of PMWIN)

Thanks for

Your attention!